

# Discrete Time Sliding-Mode Control Design with Grey Predictor

Huan-Chan Ting, Jeang-Lin Chang, Chih-Huei Yeh, and Yon-Ping Chen

## Abstract

**This paper presents an advanced GM(1,1) model which can improve the accuracy of the conventional grey prediction and applies it to discrete sliding-mode control (DSMC). Using a Lagrange polynomial to take as a compensator and combining it with the original GM(1,1) model, the proposed prediction method can decrease the prediction error and easily implement in microprocessors with less computing time and memories. Then we employ this technique in DSMC to detect the system unknown perturbation. Comparing with the conventional DSMC, the proposed algorithm can reduce the switching gain and result in that the system state is bounded in a smaller region. Numeric simulation results of a DC motor are given to illustrate the feasibility and successfulness of the proposed design.**

**Keywords:** *GM(1,1), Lagrange polynomial, sliding mode, DC motor.*

## 1. Introduction

The grey system [1], introduced by Professor Deng in 1982, is employed to deal with systems which are characterized by poor and incomplete information. The grey system theory is primarily classified into two categories: grey relational analysis and grey prediction. As a category of the grey system theory, the grey prediction is widely applied into various realms, such as earthquakes, temperature forecast [2], power demand [3], and system control [4], [5].

The first-order single-variable grey model, denoted as a GM(1,1) model, is the most popular approach for prediction and has been successfully applied to the controller design as a predictor [6-10]. Although the GM(1,1) model takes the advantages of simplicity and quickness, the limitation of its prediction accuracy is still arguable. To further improve the prediction accuracy, some investigators have proposed their modified versions of the GM(1,1) model. Hsu and Chen [2]

combined residual modification with artificial neural network sign estimation to improve the prediction accuracy. Yao, *et al* [3] added an adaptive value in the grey differential equation to improve system performance. Su, Lin, and Hsu [4] proposed a Markov-Fourier grey model, in which Fourier series is used to fit residuals and Markov matrices is employed to encode possible global information generated by residuals. In addition, some investigators [11], [12] have paid their attentions to the optimization of the background value of the GM(1,1) model in order to improve the its prediction accuracy. Actually, these modified versions only improve the prediction accuracy on a small scale, especially for a non-monotone data sequence. In practice, these above methods are very complex and hard to implement on microprocessors. To overcome this issue, a novel grey predictor combined with a Lagrange polynomial, called as an advanced GM(1,1) model, is brought up not only aiming at a more precise estimation but also giving a reasonable computation time for on-line control. The addition of the Lagrange polynomial maintains the simplicity of grey prediction and decreases the original prediction error obviously.

Discrete sliding-mode control (DSMC) [13-16], has been developed for decades, based on its robustness to system perturbations and uncertainties. In general, the strategy of DSMC is to design a surface and then force the system trajectory to approach this surface and be stabilized. However, for the system with external disturbance, the system trajectory will swing in a bounded region around the surface and decrease the performance. The main problem in the traditional DSMC design is that the width of the bounded region is dominated by the external disturbance. Although the control purpose can be achieved, the performance is bad with larger external disturbance. In addition, for suppressing this larger external disturbance, it must pay more control gain.

In this paper, a DSMC with advanced GM(1,1) model which is used to predict external disturbance is proposed. The advanced GM(1,1) model can predict the unknown bounded disturbance which is impossibly possessed by DSMC of [17]. Replacing the boundary of external disturbance in DSMC by boundary of prediction error, it can efficiently reduce the width of the bounded region to improve the system performance with lower control gain. Besides, the simplicity of the integral

---

Corresponding Author: H. C. Ting is with the Department of Electrical and Control Engineering, National Chiao-Tung University, 1001 University Rd., Hsinchu, Taiwan, 300.

E-mail: hcting.ece92g@nctu.edu.tw

Manuscript received 5 June 2007; revised 12 Sep. 2007; accepted 12 Sep. 2007.

design is more feasible to implement on real-time control.

In the next section, the fundamental configuration of grey prediction is briefly derived at first and the original GM(1,1) model is discussed. In section 3, an error compensation using a Lagrange polynomial is proposed to alleviate the prediction error of GM(1,1) model. Section 4 shows how to combine of the advanced GM(1,1) model with DSMC. Simulation results of a DC motor are given to verify the achievement of the proposed method in section 5. Finally, the conclusion is given.

### 2. Problem Statement

Giving a positive sequence, a first-order single-variable grey model, denoted as the GM(1,1) model, can be adopted to obtain the predictive data. Let the positive sequence be

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(p)\} \tag{1}$$

where  $x^{(0)}(k) > 0$ ,  $k = 1, 2, \dots, p$ , and  $p \geq 4$ . To establish the GM(1,1) model, three fundamental operations, which are the accumulated generating operation (AGO), the mean operation (MEAN) and the inverse accumulated operation (IAGO), are given by

$$\text{AGO} - x^{(1)}(k) = \sum_{j=1}^k x^{(0)}(j), \quad k = 1, 2, \dots, p, \tag{2}$$

MEAN-

$$z^{(1)}(k) = \frac{1}{2} [x^{(1)}(k) + x^{(1)}(k-1)], \quad k = 2, 3, \dots, p, \tag{3}$$

and

$$\text{IAGO} - \begin{cases} x^{(0)}(1) = x^{(1)}(1) \\ x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1), \quad k = 2, 3, \dots, p. \end{cases} \tag{4}$$

It follows from (2) and (3) that the following inequality can be obtained

$$\begin{aligned} z^{(1)}(k) - z^{(1)}(k-1) &= \frac{1}{2} [x^{(1)}(k) + x^{(1)}(k-1)] \\ &\quad - \frac{1}{2} [x^{(1)}(k-1) + x^{(1)}(k-2)] \\ &= \frac{1}{2} \left[ \sum_{j=1}^k x^{(0)}(j) - \sum_{j=1}^{k-2} x^{(0)}(j) \right] \\ &= \frac{1}{2} [x^{(0)}(k-1) + x^{(0)}(k)] > 0 \end{aligned}$$

as  $x^{(0)}(k) > 0$  for all  $k$ . According to grey system theorem [18], the GM(1,1) model is commonly constructed as the following grey differential equation

$$x^{(0)}(k) + ax^{(1)}(k) = b \quad k = 2, 3, \dots, p \tag{5}$$

where  $a$  is the development coefficient and  $b$  is the grey input. Both  $a$  and  $b$  are unknown constants which needs

to be further determined. Rewriting (5) into a matrix form yields  $y = D \begin{bmatrix} a \\ b \end{bmatrix}$  where

$$y = \begin{bmatrix} x^{(0)}(2) \\ \vdots \\ x^{(0)}(p) \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} -z^{(1)}(2) & 1 \\ \vdots & \vdots \\ -z^{(1)}(p) & 1 \end{bmatrix}. \tag{6}$$

The truth of  $z^{(1)}(k) > z^{(1)}(k-1)$  follows that  $D$  is of full rank. Based on the least square method,  $a$  and  $b$  can be solved from (6) as

$$\begin{bmatrix} a \\ b \end{bmatrix} = (D^T D)^{-1} D^T y. \tag{7}$$

By directly modifying the solution of the above equation, the term  $x^{(1)}(k)$  is estimated as [18]

$$\hat{x}^{(1)}(k) = [x^{(0)}(1) - b/a] \exp(-a(k-1)) + b/a, \tag{8}$$

where  $\hat{x}^{(1)}(k)$  is the estimation of  $x^{(1)}(k)$ . Further using IAGO in (4) yields

$$\hat{x}^{(0)}(k) = (1 - \exp(a)) \cdot [x^{(0)}(1) - b/a] \cdot \exp(-a(k-1)), \tag{9}$$

for  $k > p$ . Clearly,  $\hat{x}^{(0)}(k)$  for  $k = 2, 3, \dots$  are the so-called predictive data of the sequence (1), which can be expressed as

$$\begin{aligned} \hat{x}^{(0)}(p+j) &= (1 - \exp(a)) \\ &\quad \cdot [x^{(0)}(1) - b/a] \cdot \exp(-a(p+j-1)), \end{aligned} \tag{10}$$

where  $j = 1, 2, \dots$ . For  $j = 1$ , the one-step-ahead predictive value would be obtained by

$$\hat{x}^{(0)}(p+1) = (1 - \exp(a)) \cdot [x^{(0)}(1) - b/a] \cdot \exp(-ap) \tag{11}$$

which is the first predictive value coming after the sequence (1).

For monotone sequences, the GM(1,1) model is a suitable way to do prediction. However, the prediction precision of the GM(1,1) model will decrease if the sequence varies faster. Moreover, from (11), it is found that the exponential term is difficult to implement on hardware. For solving these two problems, an advanced GM(1,1) model is proposed to increase accuracy of the sequence prediction. The exponential terms in advanced GM(1,1) model is simplified to implement easily in microprocessors. The advanced GM(1,1) model is applied to DSMC and simulated on a DC motor.

### 3. Grey Prediction with Linear Compensation

It is well known that the exponential function  $e^x$  can be extended as

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \tag{12}$$

For simplifying the exponential calculation, the first four terms in right side of (12) are only retained to be approximate exponential function,

$$\exp(x) \approx 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}. \quad (13)$$

Because the higher order terms in (12) are omitted, the real-time operation time is shorter much. In the practice application, the exactly exponential function is not feasible and its approximation is necessary. On the other hand, the approximate calculation will increase. Hence, choosing suitable approximate order can decrease the losing accuracy in any situation. Then substituting (13) into (11), we can rewrite the GM(1,1) model as

$$\hat{x}_e(p+1) = -\left(a + \frac{a^2}{2} + \frac{a^3}{6}\right) \times \left[x^{(0)}(1) - \frac{b}{a}\right] \times \left(1 - ap + \frac{a^2 p^2}{2} - \frac{a^3 p^3}{6}\right) \quad (14)$$

where the subscript  $e$  means approximate exponential operation. The index  $p$  is number of predictive reference values of grey prediction,  $p \geq 4$ . In general case, (14) can be expressed as

$$\hat{x}_e(k+1) = -\left(a + \frac{a^2}{2} + \frac{a^3}{6}\right) \times \left[x^{(0)}(k-p+1) - \frac{b}{a}\right] \times \left(1 - ap + \frac{a^2 p^2}{2} - \frac{a^3 p^3}{6}\right). \quad (15)$$

where

$$\begin{bmatrix} a \\ b \end{bmatrix} = (\mathbf{D}^T \mathbf{D})^{-1} \mathbf{D}^T \mathbf{y}, \quad \mathbf{y} = \begin{bmatrix} x^{(0)}(k-p+2) \\ \vdots \\ x^{(0)}(k) \end{bmatrix}$$

and

$$\mathbf{D} = \begin{bmatrix} -z^{(1)}(k-p+2) & 1 \\ \vdots & \vdots \\ -z^{(1)}(k) & 1 \end{bmatrix}. \quad (16)$$

Note that the above equation is the least-square solution due to the non-square matrix  $\mathbf{D}$ . It is one of reasons caused the estimation error of grey prediction. For reducing the errors of original GM(1,1) model and approximate exponential operation, the Lagrange form of the interpolating polynomial [19] is applied to compensate the grey prediction. First, define an error function  $e(k) = x^{(0)}(k) - \hat{x}_e(k)$ . After  $p$  times grey prediction, we have the data set  $\{e(k-p+1), \dots, e(k-1), e(k)\}$  at the  $k$ th sampling point.

Using the concept of Lagrange interpolation, the compensation value of the next point is given by

$$\hat{e}(k+1) = e(k-p+1)L_1(k+1) + e(k-p+2)L_2(k+1) + \dots + e(k)L_p(k+1)$$

$$= \sum_{j=1}^p e(k-p+j)L_j(k+1). \quad (17)$$

where  $p \geq 2$  and the function  $L_j(p+1)$  is the so-called Lagrange polynomial, which is given by

$$L_j(k+1) = \prod_{i=1, i \neq j}^p \frac{k+1-i}{j-i}. \quad (18)$$

For example, if we choose the predictive reference value of grey prediction  $p = 4$ , the  $k$ th error term is estimated by

$$\hat{e}(k+1) = -e(k-3) + 4 \cdot e(k-2) - 6 \cdot e(k-1) + 4 \cdot e(k). \quad (19)$$

Hence, the advanced GM(1,1) model compensated by the Lagrange polynomial is given by

$$\hat{x}^{(0)}(k+1) = \hat{x}_e(k+1) + \hat{e}(k+1). \quad (20)$$

As the error of advanced GM(1,1) model is small, the low order Lagrange polynomial is enough to achieve the performance. However, if the prediction error is large, higher order polynomial and higher order grey model are both applied to decrease the error. Usually, higher order Lagrange polynomial is adopted because its operation is simpler than higher order grey model. Notice that the proper order compensation is more useful way not the highest order when memories, operation time and truncation error are limited.

#### 4. Discrete Sliding-Mode Controller with Grey Prediction

Consider a discrete time system

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}(\mathbf{u}(k) + \mathbf{w}(k)) \quad (21)$$

where  $\mathbf{x}(k) \in \mathfrak{R}^n$ ,  $\mathbf{u}(k) \in \mathfrak{R}^m$  and  $\mathbf{w}(k) \in \mathfrak{R}^m$  are the state vector, control input vector and matched disturbance vector, respectively. Assume that the pair  $(\mathbf{A}, \mathbf{B})$  is controllable, where  $\mathbf{A} \in \mathfrak{R}^{n \times n}$  and  $\mathbf{B} \in \mathfrak{R}^{n \times m}$  are constant matrices. We use the virtual eigenvalue method, proposed by Chang and Chen [17], to design the sliding surface. First by using the pole placement method, the feedback gain matrix  $\mathbf{F} \in \mathfrak{R}^{m \times n}$  can be calculated such that  $\mathbf{A} - \mathbf{B}\mathbf{F}$  is stable. The corresponding

eigenvector matrix  $\begin{bmatrix} \mathbf{W}_r \\ \mathbf{W}_s \end{bmatrix}$  is given by

$$\begin{bmatrix} \mathbf{W}_r \\ \mathbf{W}_s \end{bmatrix} (\mathbf{A} - \mathbf{B}\mathbf{F}) = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Psi} \end{bmatrix} \begin{bmatrix} \mathbf{W}_r \\ \mathbf{W}_s \end{bmatrix} \quad (22)$$

where  $\mathbf{W}_r \in \mathbb{C}^{(n-m) \times n}$  and  $\mathbf{W}_s \in \mathfrak{R}^{m \times n}$  are the left eigenvector matrices corresponding to eigenvalues of  $\mathbf{A} \in \mathbb{C}^{(n-m) \times (n-m)}$  and  $\mathbf{\Psi} \in \mathfrak{R}^{m \times m}$ , respectively. Both  $\mathbf{A}$  and  $\mathbf{\Psi}$  are diagonal matrices, which diagonal elements are assigned to be sliding-mode eigenvalues

$\{\lambda_1, \lambda_2, \dots, \lambda_{n-m}\}$  and virtual eigenvalues  $\{\psi_1, \psi_2, \dots, \psi_m\}$ , respectively. These virtual eigenvalues must satisfy the following conditions:

- (C1)  $\psi_i \neq \lambda_j$  for  $1 \leq i \leq m$  and  $1 \leq j \leq n-m$ .
- (C2) Any eigenvalues  $\{\psi_1, \psi_2, \dots, \psi_m\}$  is not in the spectrum of  $A$ .

In general, the virtual eigenvalue  $\psi_i$  is designed as  $0 < \psi_i < 1$  in discrete time systems,  $i=1, 2, \dots, m$ . Notice that the virtual eigenvalue  $\psi_i$  does not affect sliding-mode but can adjust the speed of the system trajectory approaching the sliding surface.

The matrix  $W_s B \in \mathbb{R}^{m \times m}$  is invertible under condition (C2). In addition, from  $W_s(A - BF) = \Psi W_s$  in (22) another important equation can be obtained as

$$\begin{aligned} (W_s B)^{-1} W_s(A - BF) &= (W_s B)^{-1} \Psi W_s \\ &= (W_s B)^{-1} \Psi W_s B (W_s B)^{-1} W_s \\ &= \Phi_s (W_s B)^{-1} W_s \end{aligned} \quad (23)$$

where  $\Phi_s = (W_s B)^{-1} \Psi W_s B \in \mathbb{R}^{m \times m}$  is a stable matrix with eigenvalues  $\{\psi_1, \psi_2, \dots, \psi_m\}$ . The sliding surface  $s(k) \in \mathbb{R}^m$  can be selected as

$$s(k) = Cx(k) \quad (24)$$

where  $C = (W_s B)^{-1} W_s$ . Multiplying  $C$  to both sides of (21), the real disturbance value at last sampling time can be given by

$$\begin{aligned} w(k-1) &= (CB)^{-1} \{Cx(k) - CAx(k-1) - CBu(k-1)\} \\ &= Cx(k) - CAx(k-1) - u(k-1). \end{aligned} \quad (25)$$

Then we use the advanced GM(1,1) model to predict the disturbance. According to the three fundamental operations of advanced GM(1,1) model, if we choose  $p=4$ , the following equations are given

$$\text{AGO: } w_i^{(1)}(l) = \sum_{j=k-4}^l w_i(j), \quad (26)$$

$$\text{MEAN: } z_i(l) = [w_i^{(1)}(l) + w_i^{(1)}(l-1)]/2, \quad (27)$$

and

$$\begin{aligned} \text{IAGO: } w_i(k-4) &= w_i^{(1)}(k-4) \\ w_i(l) &= w_i^{(1)}(l) - w_i^{(1)}(l-1) \end{aligned} \quad (28)$$

where  $l = (k-4), (k-3), (k-2), (k-1)$ ,  $i=1, 2, \dots, m$ .

Using (15), we have

$$\begin{aligned} \hat{w}_{ei}(k) &= -\left(a_i + \frac{a_i^2}{2} + \frac{a_i^3}{6}\right) \times \left(w_i(k-4) - \frac{b_i}{a_i}\right) \\ &\quad \times \left(1 - a_i(k-1) + \frac{a_i^2(k-1)^2}{2} - \frac{a_i^3(k-1)^3}{6}\right). \end{aligned} \quad (29)$$

The coefficients  $a_i$  and  $b_i$  are calculated by

$$\begin{bmatrix} a_i \\ b_i \end{bmatrix} = (D_i^T D_i)^{-1} D_i^T y_i \quad (30)$$

where

$$y_i = \begin{bmatrix} w_i(k-3) \\ w_i(k-2) \\ w_i(k-1) \end{bmatrix} \text{ and } D_i = \begin{bmatrix} -z_i(k-3) & 1 \\ -z_i(k-2) & 1 \\ -z_i(k-1) & 1 \end{bmatrix}. \quad (31)$$

Let the approximated exponential operation error be  $e_i(k-1) = w_i(k-1) - \hat{w}_{ei}(k-1)$ . From (19), we have

$$\begin{aligned} \hat{e}_i(k) &= -e_i(k-4) + 4e_i(k-3) \\ &\quad - 6e_i(k-2) + 4e_i(k-1) \end{aligned} \quad (32)$$

Hence the advanced prediction vector is given by

$$\hat{w}(k) = [\hat{w}_1(k) \quad \hat{w}_2(k) \quad \dots \quad \hat{w}_m(k)]^T, \quad (33)$$

where  $\hat{w}_i(k) = \hat{w}_{ei}(k) + \hat{e}_i(k)$ . Let the prediction error  $\tilde{w}(k) = w(k) - \hat{w}(k)$  be bounded. It follows that  $\|\tilde{w}(k)\|_2 < \delta$  where  $\delta$  is a known constant.

Chen and Chang [17] proposed a two-layer DSMC design to deal with a system having matched and bounded disturbance. The two-layer design can avoid the high gain control in the conventional DSMC design. Applying the above advanced grey prediction of disturbance to the two-layer design method, the control law can be designed as

$$u(k) = -Fx(k) - \hat{w}(k) - \Psi \Sigma \text{sat}(s(k), \sigma) \quad (34)$$

where  $\Sigma \in \mathbb{R}^{m \times m}$  is a diagonal matrix where the diagonal elements are all  $\sigma$  and the saturation function  $\text{sat}(s_i(k), \sigma)$  is given by

$$\text{sat}(s_i(k), \sigma) = \begin{cases} \text{sgn}(s_i(k)), & |s_i(k)| > \sigma \\ s_i(k)/\sigma, & |s_i(k)| \leq \sigma. \end{cases} \quad (35)$$

The parameter  $\sigma$  is designed as  $\sigma = \frac{\delta + \varepsilon}{\psi_i} > \delta$  where

$\varepsilon > 0$  is a constant. Substituting (35) into (22), the state equation can be rewritten as

$$\begin{aligned} x(k+1) &= (A - BF)x(k) \\ &\quad + B(w(k) - \hat{w}(k) - \Psi \Sigma \text{sat}(s(k), \sigma)). \end{aligned} \quad (36)$$

Multiply  $C$  to both side of (36) to obtain

$$s(k+1) = \Psi s(k) + \tilde{w}(k) - \Psi \Sigma \text{sat}(s(k), \sigma) \quad (37)$$

or

$$s_i(k+1) = \psi_i s_i(k) + \tilde{w}_i(k) - \psi_i \sigma \text{sat}(s_i(k), \sigma). \quad (38)$$

The behavior of  $s_i(k)$  is analyzed by the following theorem.

*Theorem.* Given system (21), if the control input is given by (34) and  $C$  is designed as (24), then the sliding

surface enters the approaching layer  $|s_i(k)| \leq \sigma$  in finite steps and is finally constrained in the sliding layer  $|s_i(k)| < \delta$ .

*Proof:* Consider the following two cases;

Case I: When  $|s_i(k)| > \sigma$ , (38) can be rewritten as

$$\begin{aligned} s_i(k+1) &= \psi_i s_i(k) + \tilde{w}_i(k) - \psi_i \sigma \operatorname{sgn}(s_i(k)) \\ &= \psi_i (|s_i(k)| - \sigma) \operatorname{sgn}(s_i(k)) + \tilde{w}_i(k) \end{aligned} \quad (39)$$

and

$$\begin{aligned} |s_i(k+1)| &\leq |\psi_i (|s_i(k)| - \sigma) \operatorname{sgn}(s_i(k))| + |\tilde{w}_i(k)| \\ &< \psi_i (|s_i(k)| - \sigma) + \delta \\ &< \psi_i |s_i(k)| - \psi_i \sigma + \delta \\ &< \psi_i |s_i(k)| - \varepsilon \end{aligned} \quad (40)$$

where  $0 < \psi_i < 1$  and  $\varepsilon > 0$ . The above inequality (40) shows that the sliding surface enters the so-called approaching layer  $|s_i(k)| \leq \sigma$  in finite steps. Note that the approaching speed can be adjust by the parameter  $\psi_i$ .

Case II: When  $|s_i(k)|$  is in the approaching layer  $|s_i(k)| \leq \sigma$ , we can obtain the following relationship

$$s_i(k+1) = \psi_i s_i(k) + \tilde{w}_i(k) - \psi_i \sigma \cdot s_i(k) / \sigma = \tilde{w}_i(k)$$

and

$$|s_i(k+1)| < \delta. \quad (41)$$

Hence  $|s_i(k)|$  indeed enters the approaching layer in finite steps and is finally constrained in the sliding layer  $|s_i(k)| < \delta$ . The proof is completed.

If the approaching layer is designed as  $\sigma = \delta / \psi_i$ , the convergence of the sliding surface is still guaranteed. In practice, the upper bound of prediction error  $\delta$  can not be obtained exactly. For holding the convergence of the sliding surface, we add  $\varepsilon$  to dominate the thickness of the approaching layer.

## 5. Simulation Results

Consider a continuous time DC motor [20] where its discrete time model is

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}(u(k) + w(k)).$$

where

$$\mathbf{A} = \begin{bmatrix} -0.0005 & -0.0022 & 0 \\ 0.2412 & 0.9642 & 0 \\ 0.0024 & 0.0098 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 0.0324 \\ 0.4012 \\ 0.002 \end{bmatrix}.$$

Let  $w(k) = 0.1 \cos(0.20k) + 0.05 \sin(0.46k)$  and design

$\mathbf{f} = [-13.7194 \quad 0.0696 \quad 0.4276]$ ,  $\mathbf{A} = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.93 \end{bmatrix}$ , and  $\psi_1 = 0.05$  such that  $\mathbf{A} - \mathbf{b}\mathbf{f}$  is stable. The vector  $\mathbf{c} = [27.5388 \quad 0.2612 \quad 0.8552]$  is calculated by (25). The controller parameter  $\varepsilon$  is chosen as 0.021. The initial condition is set as  $\mathbf{x}(0) = [0.8 \quad 0.1 \quad 0.1]^T$ .

In the DC motor case, the second order Lagrange polynomial compensation is enough to achieve the prediction performance and hence is selected. The higher order compensation maybe increases little prediction performance but brings large computing time and memories cost. First, two trajectories of the state norms, which controlled by DSMC with and without advanced GM(1,1), are depicted in Figure 1. It is easy to see the better performance which controlled by DSMC with advanced GM(1,1). Figure 2 shows the prediction errors with advanced GM(1,1) and GM(1,1) model. It is obviously verify that the prediction performance of advanced GM(1,1) model is indeed more than GM(1,1) model. The trajectories of  $\mathbf{x}(k)$  are depicted in Figures 3 to 5. With GM(1,1) model, unless  $x_1$  approaches to zero,  $x_2$  and  $x_3$  are not suppressed to zero. On the other hand, all states with the advanced GM(1,1) model approach to zero. The sliding surface  $s(k)$  is shown in Figures 6 and 7. The system trajectory exponentially enters the approaching layer in finite steps and is finally bounded in the sliding layer in the case of advanced GM(1,1) model, but GM(1,1) model doesn't have the phenomenon. The control inputs in both cases are depicted in Figure 8. The control input with advanced GM(1,1) model successfully force the system to be stable and its magnitude is quite small. Based on these figures, applying the proposed control law and advanced GM(1,1) model to real cases, e.g. DC motors, is feasible and reliable.

## 6. Conclusion

The advanced GM(1,1) model, compensated by a Lagrange polynomial, is proposed and indeed decreases the prediction error. Simulation results indicate that the proposed prediction indeed improve the accuracy of the original GM(1,1) model. Using the prediction technique of the advanced GM(1,1) model in DSMC, it follows that the controller needs only a lower switching gain and the system performance is better than the conventional design method. The proposed control law is successfully feasible by simulation results of a DC motor.

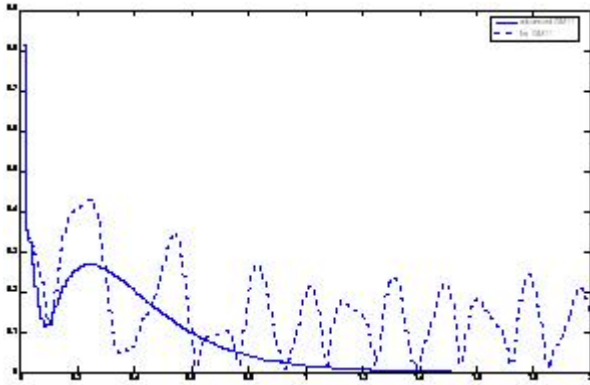


Figure 1. The norm of state  $\|x\|$ .

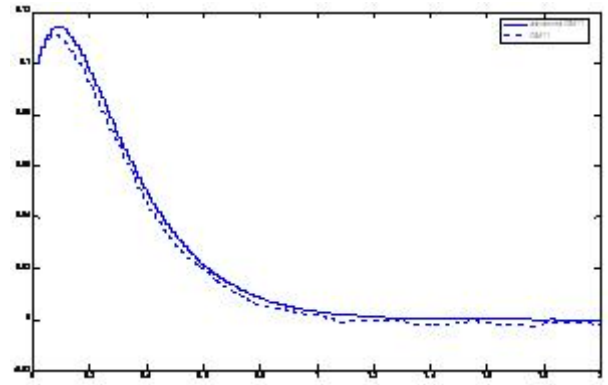


Figure 5. The trajectory of  $x_3$ .

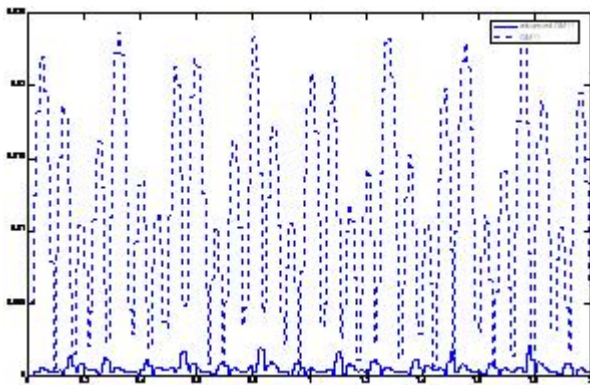


Figure 2. The prediction error.

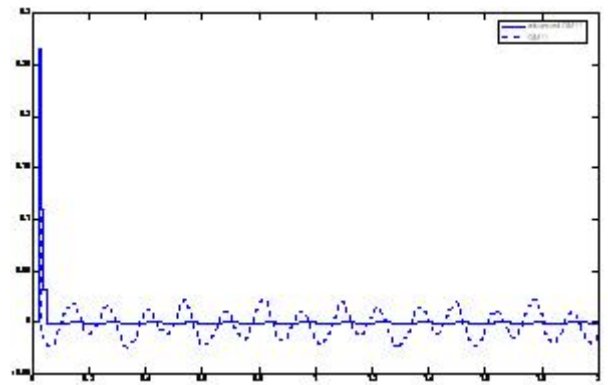


Figure 6. The sliding function  $s$ .

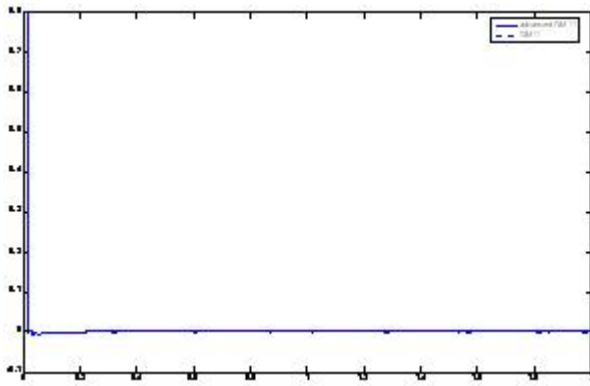


Figure 3. The trajectory of  $x_1$ .

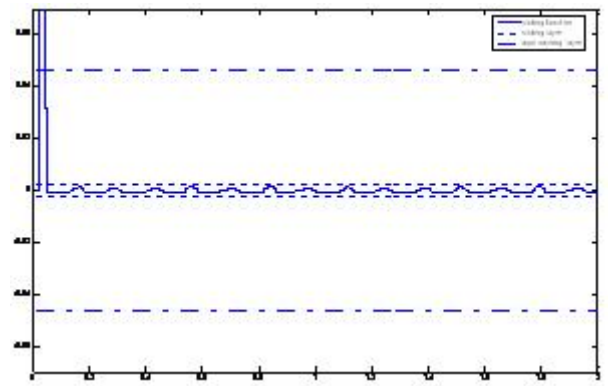


Figure 7. The trajectory of  $s$  with advanced GM(1,1) model in the pseudo-sliding layer.

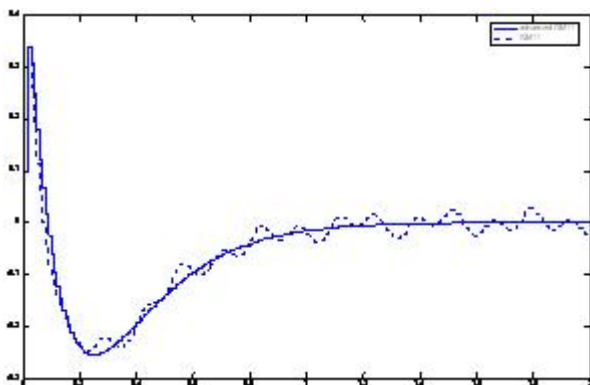


Figure 4. The trajectory of  $x_2$ .

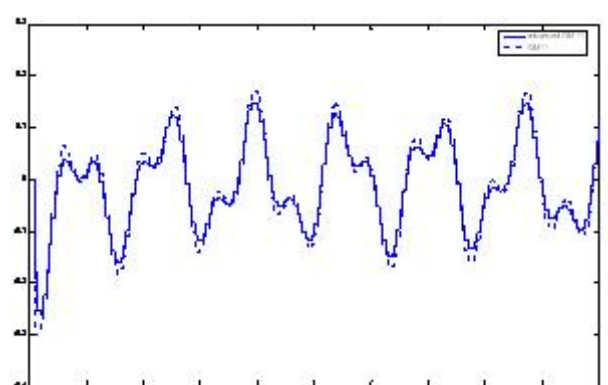


Figure 8. The control input  $u$ .

## 7. References

- [1] J. L. Deng, "Control problems of grey systems," *System & Control Letters*, vol. 1, pp. 288-294, 1982.
- [2] C. C. Hsu and C. Y. Chen, "Applications of improved grey prediction model for power demand forecasting," *Energy Conversion and Management*, vol. 44, pp. 2241-2249, 2003.
- [3] W. L. Yao, S. C. Chi, and J. H. Chen, "An improved grey-based approach for electricity demand forecasting," *Electric Power Systems Research*, vol. 67, Issue 3, pp. 217-224, Dec. 2003.
- [4] S. F. Su, C. B. Lin, and Y. T. Hsu, "A High Precision Global Prediction Approach Based on Local Prediction Approaches," *IEEE Trans. on Syst. Man Cybern. – Part C: Application and Reviews*, vol. 32, no. 4, pp. 416-425, Nov. 2002.
- [5] Z. L. Wang and S. F. Liu, "Trend Testing of Grey Dynamic Models," In *IEEE International Conf. on Syst. Man Cybern.*, vol. 3, pp. 2438-2442, Oct. 2004.
- [6] C. C. Wong and C. C. Chen, "Gain Scheduling of Grey Prediction Fuzzy Control Systems," *International Journal of Fuzzy Systems*, vol. 2, no. 3, pp. 198-204, 2000.
- [7] Y. H. Lin and T. H. Hung, "Sliding Mode Control via Grey Prediction," *The Journal of Grey System*, vol. 8, no. 4, pp. 359-380, 1996.
- [8] C. H. Chou, "A Variable Structure Controller Based on the Grey Prediction Technology," In *Proceedings of the American Control Conference*, pp. 1505-1506, 2001.
- [9] K. T. Li and Y. P. Chen, "Sliding-Mode Controller Design with Grey Prediction for Matched Disturbance," *International Journal of Fuzzy Systems*, vol. 6, no. 1, pp. 1-8, 2004.
- [10] M. F. Yeh and H. C. Lu, "A New Modified Grey Model," *The Journal of Grey System*, vol. 8, no. 3, pp. 209-216, 1996.
- [11] K. L. Wen, T. C. Chang, H. T. Chang, and M. L. You, "The Adaptive  $\alpha$  in GM(1,1) Model," In *IEEE SMC International Conference*, pp. 304-308, 1999.
- [12] B. R. Chang, "A Profile of Residual Error Fluctuation for GM(1,1|  $\alpha$ ) Grey Prediction Model," *International Journal of Fuzzy Systems*, vol. 4, no. 3, pp. 816-825, 2002.
- [13] K. Furuta, "Sliding mode control of a discrete system," *System & Control Letters*, vol. 4, pp. 145-152, 1990.
- [14] C. Milosavljevic, "General conditions for the existence of a quasisliding mode on the switching hyperplane in discrete variable structure systems," *Automatic Remote Control*, vol. 46, pp. 307-314, 1985.
- [15] S. Z. Sarpturk, Y. I Stefanopoulos, and O. Kaynak, "On the Stability of Discrete-Time Sliding Mode Control Systems," *IEEE Trans. on Automatic Control*, vol. AC-32, no. 10, pp. 930-932, Oct. 1987.
- [16] S. K. Spurgeon, "Hyperplane design techniques for discrete-time variable structure control systems," *International Journal of Control*, vol. 55, no. 2, pp. 445-456, 1992.
- [17] Y. P. Chen and J. L. Chang, "On Discrete Pseudo-Sliding Control," *Journal of Control System and Technology*, vol. 6, no. 4, pp. 253-257, 1998.
- [18] J. L. Deng, "Introduction of grey system theory," *The Journal of Grey System*, vol. 1, no. 1, pp. 1-24, 1989.
- [19] C. J. Zarowski, *An Introduction to Numerical Analysis for Electrical and Computer Engineers*, New Jersey: Wiley, 2004.
- [20] B. C. Kuo and F. Golnaraghi, *Automatic Control Systems*, New Jersey: Wiley, 2002.