Journal of System Design and Dynamics

Vol. 2, No. 4, 2008

Robust Discrete-Time Output Tracking Controller Design for Nonminimum Phase Systems*

Jeang-Lin CHANG**, Huan-Chan TING*** and Yon-Ping CHEN***

** Department of Electrical Engineering, Oriental Institute of Technology
Pan-Chiao, Taipei County 220, Taiwan

*** Department of Electrical and Control Engineering, National Chiao-Tung University
Hsinchu 300, Taiwan

E-mail:hcting.ece92g@nctu.edu.tw

Abstract

In regard to the nonminimum phase MIMO discrete time systems, a method using estimation method in designing an observer-based output tracking controller is proposed in this paper. Provided the variation of the disturbance in the two consecutive sampling instances is not changed significantly, both the system state and the unknown disturbance can be simultaneously estimated by our proposed observer algorithm with the estimation error being constrained in a small bounded region of the order of $O(T)$. The control law including a feedforward term and a feedback input can cause the tracking error to be bounded in a small region with the guarantied system stability. A numerical example is presented to demonstrate the applicability of the proposed control scheme.

Key words: Discrete-Time, Nonminimum Phase, State Estimator, Disturbance Observer, Output Tracking

1. Introduction

In servo systems, the tracking control, in which the control object must be moved along a desired trajectory, is a fundamental problem. For minimum phase systems, Jemaa and Davison(12) have shown that the perfect tracking can be achieved by adding proper feedforward actions, which are the inverse dynamics of the controlled plant. However, causal perfect tracking is not possible in nonminimum phase systems because the inverse dynamics becomes unstable(18). Hence, output tracking control in nonminimum phase system is a challenging problem, and it has been recently studied extensively. This problem in the discrete-time case is particularly interesting because using a zero-order-hold for a continuous-time minimum phase system may produce a sampled system with unstable zeros(2). In addition, the perturbation and state estimation, where they are various formulations, are also important for servo control. The disturbance observer method(9) is known to effectively compensate for disturbance(13). An unknown input observer(6),(10) has been developed to estimate the system state when unknown disturbance exists. In order to obtain perfect estimation, an important constraint resulting from the two above-mentioned

*Received 1 Feb., 2008 (No. 08-0059)
[DOI: 10.1299/jsdd.2.950]

methods is that the system should be minimum phase (with respect to the relation between the output and the disturbance). Hence, for nonminimum phase systems with unknown perturbations, tracking controller and estimation design are difficult but worth studying.

For a discrete-time plant with unstable zeros, the tracking performance is effective through using preview or look-ahead controller with respect to which the information of the reference signal is given in advance. Tomizuka(19) used preview to implement zero-phase error tracking control, which was based on a feedforward controller that ensures zero phase shift between the reference and the output. On the other hand, this method was proved to produce a large gain error in tracking the high frequency signal. Haack and Tomizuka(8), and Xia and Menq(20) proposed different non-causal compensators to reduce the large gain error. A non-causal scheme based on a series expansion of the unstable zeros was adopted(11). Moreover, Marconi *et al.*(15) proposed steering along zeros control, which is more general than Gross's method. However, the effect of disturbances or uncertainties was not considered in these papers and their methods are only implemented in SISO systems.

In this paper, we propose an observer-based output tracking controller for nonminimum phase MIMO systems. A proportional integral observer (PIO) structure, which uses an additionally introduced integral term of the output estimation error in the observer design can offer certain degrees of freedom, and can simultaneously estimate both the system state and the unknown perturbation. Beale and Shafai(3) used this additional freedom in the observer-based controller design, as a result of which PIO becomes less sensitive to parameter variations of the system. Shafai *et al.*(16) used the PIO to study the loop transfer recovery (LTR) problem in discrete-time systems. Moreover, Busawon and Kabore(5) demonstrated that the PIO can effectively reduce the effect of measurement noises as opposed to the proportional observer. The reason of utilizing this PIO to estimate both the state and the disturbance can be attributed to the fact that this scheme can be implemented on specific nonminimum phase systems that do not have unstable zeros lying on one. The structure and the estimation capacities of this proposed observer are discussed and analyzed, and the conditions are demonstrated. Moreover, this algorithm in digital implementations can make both the state and disturbance estimation error approaching at least to the size of $O(T)$, where T is the sampling period. Then, the command tracker generator technology is used to produce the desired reference input. Based on these estimation signals, a controller combining a PI feedback control and a feedforward input generated by the reference model is established such that the system output can approximately track the reference with a small tracking error being restrained.

This paper is organized as follows. The system description and the problem formulation are given in Section 2. Also Sections 3 and 4 present the observer and controller designs, respectively. The effectiveness of the proposed controller is illustrated in Section 5 with a numerical example. Concluding remarks are given in Section 6.

2. Problem Formulation

Consider a continuous-time square MIMO linear system described by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{H}\boldsymbol{x}(t) + \boldsymbol{D}\left(\boldsymbol{u}(t) + \boldsymbol{d}(t)\right)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)\end{aligned} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{u}, \boldsymbol{d}, \boldsymbol{y} \in \mathbb{R}^m$ are the system state, control input, matched disturbance, and output of the system, respectively. Zero-order-hold is used within the aforementioned continuous time model, $\boldsymbol{u}(t) = \boldsymbol{u}(kT)$, $kT \le t < (k+1)T$ where $k \ge 0$ is an integer. Denoting $\boldsymbol{x}(k) = \boldsymbol{x}(kT)$, $\boldsymbol{y}(k) = \boldsymbol{y}(kT)$, and $\boldsymbol{d}(k) = \boldsymbol{d}(kT)$, the discrete-time model can be given by

$$\begin{aligned}\boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k) + \boldsymbol{f}(k)\\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k)\end{aligned} \tag{2}$$

where

$$\boldsymbol{A} = \exp\left(\boldsymbol{H}T\right) \quad \text{and} \quad \boldsymbol{B} = \boldsymbol{D}T + \frac{1}{2!}\boldsymbol{H}\boldsymbol{D}T^2 + \cdots \in O\left(T\right). \tag{3}$$

Moreover, the disturbance term is given by

$$\boldsymbol{f}(k) = \int_0^T e^{\boldsymbol{H}\tau}\boldsymbol{D}\boldsymbol{d}\left((k+1)T - \tau\right)d\tau \in \mathbb{R}^n \tag{4}$$

where the magnitude of $\boldsymbol{f}$ is said to be $\boldsymbol{f} \in O\left(T^n\right)$ if

$$\lim_{T\to 0}\frac{\boldsymbol{f}}{T^n} \ne 0 \quad \text{and} \quad \lim_{T\to 0}\frac{\boldsymbol{f}}{T^{n-1}} = 0 \tag{5}$$

where n is a integer and $O\left(T^0\right) = O\left(1\right)$. In terms of discrete-time nonminimum phase systems, there exists a certain degree of substantial difficulties and challenges to design output tracking controller. Under the conditions that not only disturbance is bounded and smooth but T is also small, a design combining a state and a state-space disturbance observer is proposed to ensure that the output can track the reference signal with the tracking error being constrained in a small region. The main feature is its application in a nonminimum phase system. Before introducing the main results, the following assumptions are made throughout this paper.

(**A1**)**:** System (2) is observable and controllable.

(**A2**)**:** The condition $rank\left(\begin{bmatrix}\boldsymbol{A}-\boldsymbol{I}_n & \boldsymbol{B}\\ \boldsymbol{C} & \boldsymbol{0}\end{bmatrix}\right) = n+m$ is satisfied and $\left|\boldsymbol{C}\boldsymbol{B}\right| \ne 0$. This condition implies that system (2) has no transmission zeros at one and it rises in response to the robust servomechanism problem(12).

(**A3**) (1)**:** The sampling interval T is chosen such that the disturbance does not vary too much between two consecutive sampling instances. Moreover, the variation of the disturbance between two consecutive sampling instances is bounded, i.e., $\boldsymbol{d}(k+1) - \boldsymbol{d}(k) \in O\left(T\right)$.

The purpose of presuming three assumptions is that with respect to system (2) a designed controller is proposed to bring surely the system output to be as close as possible to the reference signal.

3. State estimator and disturbance observer

Since the characteristics of system are affected by unknown disturbance, it is our emphasis in this section to propose an unprecedented estimator. Concurrently, both state and unknown disturbance can all be estimated effectively. By comparison, the unknown input disturbance developed by other predecessors can be implemented in minimum phase systems only. However, our proposed method can be used in nonminimum phase systems. The characteristics and performance of this estimator is to be represented by Theorem 1. Although our method cannot reach to its perfection in performing estimation, the estimation error is restricted to a less value than $O(T)$.

In this section, an efficient algorithm is proposed to use previous $\boldsymbol{y}(k)$ attained in predicting $\boldsymbol{x}(k)$ and $\boldsymbol{f}(k)$ without requiring no other information on $\boldsymbol{f}(k)$. Let $\hat{\boldsymbol{x}}(k)$ and $\hat{\boldsymbol{y}}(k)$ be the estimations of $\boldsymbol{x}(k)$ and $\boldsymbol{y}(k)$, respectively. Design a state and a disturbance observer as

$$\begin{aligned}
&\hat{\boldsymbol{x}}(k+1) = \boldsymbol{A}\hat{\boldsymbol{x}}(k) + \boldsymbol{B}\boldsymbol{u}(k) + \boldsymbol{F}\left(\boldsymbol{y}(\boldsymbol{k}) - \hat{\boldsymbol{y}}(\boldsymbol{k})\right) + \boldsymbol{B}\left(\boldsymbol{L}_2\boldsymbol{p}_1(k) + \boldsymbol{L}_1\boldsymbol{p}_2(k)\right)\\
&\boldsymbol{p}_1(k+1) = \boldsymbol{p}_1(k) + \boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k)\\
&\boldsymbol{p}_2(k+1) = \boldsymbol{p}_2(k) + \boldsymbol{p}_1(k)\\
&\hat{\boldsymbol{y}}(k) = \boldsymbol{C}\hat{\boldsymbol{x}}(k)
\end{aligned} \tag{6}$$

where $\boldsymbol{F} \in \mathbb{R}^{n\times m}$, and $\boldsymbol{L}_i \in \mathbb{R}^{m\times m}$ for $1 \le i \le 2$ are parameter matrices designed latter. Denote $\tilde{\boldsymbol{x}}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$ and $\tilde{\boldsymbol{y}}(k) = \boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k)$ as the estimation errors of the state and the output, respectively.

Lemma 1: Suppose that $\boldsymbol{d}(t)$ is bounded and smooth and (A3) holds, then $\boldsymbol{f}(k)$ and $\boldsymbol{d}(k)$ satisfy the following equations:

$$\boldsymbol{f}(k) = \boldsymbol{B}\boldsymbol{d}(k) + O\left(T^2\right), \tag{7}$$

$$\boldsymbol{d}(k+1) - \boldsymbol{d}(k) \in O(T) \text{ and } \boldsymbol{d}(k+2) - 2\boldsymbol{d}(k+1) + \boldsymbol{d}(k) \in O\left(T^2\right). \tag{8}$$

It is worth noting that due to the significance of sampling the disturbance matched originally in continuous time systems can be altered into a state of mismatched disturbance in discrete time systems.

Proof: See Ref. (1). □

Applying Lemma 1 to Eq. (7) yields

$$\tilde{\boldsymbol{x}}(k+1) = \left(\boldsymbol{A} - \boldsymbol{F}\boldsymbol{C}\right)\tilde{\boldsymbol{x}}(k) - \boldsymbol{B}\left(\boldsymbol{L}_2\boldsymbol{p}_1(k) + \boldsymbol{L}_1\boldsymbol{p}_2(k)\right) + \boldsymbol{B}\boldsymbol{d}(k) + O\left(T^2\right). \tag{9}$$

Define two new variables

$$\boldsymbol{w}_1(k) = \boldsymbol{d}(k+1) - \boldsymbol{d}(k) - \boldsymbol{L}_1\boldsymbol{p}_1(k) \tag{10}$$

and

$$\boldsymbol{w}_2(k) = \boldsymbol{d}(k) - \boldsymbol{L}_2\boldsymbol{p}_1(k) - \boldsymbol{L}_1\boldsymbol{p}_2(k) \tag{11}$$

where $\boldsymbol{w}_1 \in \mathbb{R}^m$ and $\boldsymbol{w}_2 \in \mathbb{R}^m$. The dynamics equations of $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ are given by

$$\begin{aligned}
\boldsymbol{w}_1(k+1) &= \boldsymbol{d}(k+2) - \boldsymbol{d}(k+1) - \boldsymbol{L}_1\boldsymbol{p}_1(k+1)\\
&= \boldsymbol{d}(k+2) - \boldsymbol{d}(k+1) - \boldsymbol{L}_1\left(\boldsymbol{p}_1(k) + \tilde{\boldsymbol{y}}(k)\right)
\end{aligned}$$

$$= \boldsymbol{d}(k+2) - \boldsymbol{d}(k+1) - \left(\boldsymbol{d}(k+1) - \boldsymbol{d}(k) - \boldsymbol{w}_1(k)\right) - \boldsymbol{L}_1\tilde{\boldsymbol{y}}(k)$$
$$= \boldsymbol{w}_1(k) + \boldsymbol{d}(k+2) - 2\boldsymbol{d}(k+1) + \boldsymbol{d}(k) - \boldsymbol{L}_1\tilde{\boldsymbol{y}}(k) \tag{12}$$

and

$$\begin{aligned} \boldsymbol{w}_2(k+1) &= \boldsymbol{d}(k+1) - \boldsymbol{L}_2\boldsymbol{p}_1(k+1) - \boldsymbol{L}_1\boldsymbol{p}_2(k+1) \\ &= \boldsymbol{d}(k+1) - \boldsymbol{L}_1\boldsymbol{p}_1(k) - \left(\boldsymbol{L}_2\boldsymbol{p}_1(k) + \boldsymbol{L}_1\boldsymbol{p}_2(k)\right) - \boldsymbol{L}_2\tilde{\boldsymbol{y}}(k) \\ &= \boldsymbol{w}_1(k) + \boldsymbol{d}(k) - \left(\boldsymbol{d}(k) - \boldsymbol{w}_2(k)\right) - \boldsymbol{L}_2\tilde{\boldsymbol{y}}(k) \\ &= \boldsymbol{w}_1(k) + \boldsymbol{w}_2(k) - \boldsymbol{L}_2\tilde{\boldsymbol{y}}(k). \end{aligned} \tag{13}$$

It follows from Eqs. (9) and (11) that

$$\tilde{\boldsymbol{x}}(k+1) = \left(\boldsymbol{A} - \boldsymbol{F}\boldsymbol{C}\right)\tilde{\boldsymbol{x}}(k) + \boldsymbol{B}\boldsymbol{w}_2(k) + O\left(T^2\right). \tag{14}$$

Let $\boldsymbol{w}(k) = \left[\tilde{\boldsymbol{x}}^T(k) \quad \boldsymbol{w}_1^T(k) \quad \boldsymbol{w}_2^T(k)\right]^T \in \mathbb{R}^{n+2m}$ and augment Eqs. (12) to (14) in matrix form

$$\begin{aligned} \boldsymbol{w}(k+1) &= \begin{bmatrix} \boldsymbol{A}-\boldsymbol{F}\boldsymbol{C} & \boldsymbol{0} & \boldsymbol{B} \\ -\boldsymbol{L}_1\boldsymbol{C} & \boldsymbol{I}_m & \boldsymbol{0} \\ -\boldsymbol{L}_2\boldsymbol{C} & \boldsymbol{I}_m & \boldsymbol{I}_m \end{bmatrix} \boldsymbol{w}(k) + \begin{bmatrix} O\left(T^2\right) \\ \boldsymbol{d}(k+2) - 2\boldsymbol{d}(k+1) + \boldsymbol{d}(k) \\ \boldsymbol{0} \end{bmatrix} \\ &= \left(\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\right)\boldsymbol{w}(k) + O\left(T^2\right) \\ \tilde{\boldsymbol{y}}(k) &= \left[\boldsymbol{C} \quad \boldsymbol{0} \quad \boldsymbol{0}\right]\boldsymbol{w}(k) = \bar{\boldsymbol{C}}\boldsymbol{w}(k) \end{aligned} \tag{15}$$

where $\bar{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} & \boldsymbol{B} \\ \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{I}_m \end{bmatrix}$, $\boldsymbol{L} = \left[\boldsymbol{F}^T \quad \boldsymbol{L}_1^T \quad \boldsymbol{L}_2^T\right]^T$, and $\bar{\boldsymbol{C}} = \left[\boldsymbol{C} \quad \boldsymbol{0}_m \quad \boldsymbol{0}_m\right]$. From the above, if $\left(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}\right)$ is observable, $\boldsymbol{L}$ would be found such that $\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}$ is stable.

Lemma 2: If $\left(\boldsymbol{A}, \boldsymbol{C}\right)$ is observable and (A2) holds, then $\left(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}\right)$ is observable.

Proof: From the linear system, the pair $\left(\boldsymbol{A}, \boldsymbol{C}\right)$ is observable if and only if $\operatorname{rank}\left(\begin{bmatrix} \lambda\boldsymbol{I}_n - \boldsymbol{A} \\ \boldsymbol{C} \end{bmatrix}\right) = n$, $\forall\ \lambda \in C$. Hence, the pair $\left(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}\right)$ is observable if and only if

$$\operatorname{rank}\left(\begin{bmatrix} \lambda\boldsymbol{I}_{n+2m} - \bar{\boldsymbol{A}} \\ \bar{\boldsymbol{C}} \end{bmatrix}\right) = \operatorname{rank}\left(\begin{bmatrix} \lambda\boldsymbol{I}_n - \boldsymbol{A} & \boldsymbol{0} & \boldsymbol{B} \\ \boldsymbol{0} & (\lambda-1)\boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_m & (\lambda-1)\boldsymbol{I}_m \\ \boldsymbol{C} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\right) = n + 2m \quad \forall\ \lambda \in C.$$

We discuss the two cases, (i) $\lambda \neq 1$ and (ii) $\lambda = 1$ in the following.

(i) When $\lambda \neq 1$, we have that $\left(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}}\right)$ is observable due to the observable pair $\left(\boldsymbol{A}, \boldsymbol{C}\right)$.

(ii) When $\lambda = 1$, it follows that $\operatorname{rank}\left(\begin{bmatrix} \lambda\boldsymbol{I}_{n+2m} - \bar{\boldsymbol{A}} \\ \bar{\boldsymbol{C}} \end{bmatrix}\right) = m + \operatorname{rank}\left(\begin{bmatrix} \boldsymbol{I}_n - \boldsymbol{A} & -\boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix}\right)$.

From (A2), one can obtain $\operatorname{rank}\left(\begin{bmatrix} \lambda\boldsymbol{I}_{n+2m} - \bar{\boldsymbol{A}} \\ \bar{\boldsymbol{C}} \end{bmatrix}\right) = m + n + m = n + 2m$. This completes the proof of the lemma. □

From Eqs. (6) and (8), it can be found that the estimation disturbance is

$$\hat{\boldsymbol{d}}(k) = \boldsymbol{L}_2 \boldsymbol{p}_1(k) + \boldsymbol{L}_1 \boldsymbol{p}_2(k) \tag{16}$$

whereas $\boldsymbol{w}_2 = \boldsymbol{d} - \hat{\boldsymbol{d}}$ represent the estimation error of the disturbance. As the proposed scheme implies, the performance of this estimator satisfies the property of Theorem 1.

Theorem 1: Consider the system (2) and the estimator (6). If these (A1)-(A3) hold and the disturbance is smooth enough, then the estimation errors will be bounded by

$$\lim_{k\to\infty} \|\tilde{\boldsymbol{x}}(k)\| \le O(T) \ \text{ and } \ \lim_{k\to\infty} \|\boldsymbol{w}_2(k)\| \le O(T).$$

Proof: We rewrite Eq. (9) as

$$\boldsymbol{w}(k+1) = \left(\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\right)\boldsymbol{w}(k) + \boldsymbol{h}(k) \tag{17}$$

where $\|\boldsymbol{h}(k)\| \in O(T^2)$. It follows from Eq. (17) that

$$\|\boldsymbol{w}(k)\| \le \left\|\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\right\|^k \|\boldsymbol{w}(0)\| + \frac{1 - \left\|\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\right\|^k}{1 - \left\|\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\right\|} \|\boldsymbol{h}(k)\|.$$

Since $\boldsymbol{L}$ from Lemma 2 would be found such that that $\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}$ is stable, as $k \to \infty$, the norm of $\boldsymbol{w}(k)$ is bounded by $\lim_{k\to\infty} \|\boldsymbol{w}(k)\| \le \frac{\|\boldsymbol{h}(k)\|}{1 - \left\|\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\right\|}$. Using Tustin's approximation(1), we have $1 - \left\|\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\right\| = 1 - \lambda_{\max}\left(\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\right) = 1 - \frac{2+Tp}{2-Tp} = \frac{-2Tp}{2-Tp} \in O(T)$, where $\lambda_{\max}$ is the maximum eigenvalue and p is the corresponding eigenvalue in continuous-time and is assumed to be $O(1)$. Hence, $\lim_{k\to\infty} \|\boldsymbol{w}(k)\| \le O(T^{-1})O(T^2) = O(T)$ and we have $\lim_{k\to\infty} \|\tilde{\boldsymbol{x}}(k)\| \le O(T)$ and $\lim_{k\to\infty} \|\boldsymbol{w}_2(k)\| \le O(T)$. The proof of this theorem is complete. □

Remark 1: If the conventional Luenberger observer is used to estimate the system state:

$$\hat{\boldsymbol{x}}(k+1) = \boldsymbol{A}\hat{\boldsymbol{x}}(k) + \boldsymbol{B}\boldsymbol{u}(k) + \boldsymbol{L}\left(\boldsymbol{y}(k) - \boldsymbol{C}\hat{\boldsymbol{x}}(k)\right)$$

where $\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C}$ is stable. From Eq. (2), we have $\tilde{\boldsymbol{x}}(k+1) = \left(\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C}\right)\tilde{\boldsymbol{x}}(k) + \boldsymbol{f}(k)$. It follows that the estimation error of system state can be bounded, $\lim_{k\to\infty} \|\tilde{\boldsymbol{x}}(k)\| \le O(1)$, similar to that of the illustrative work demonstrated by Theorem 1. Therefore, the proposed method can attain much more accurate estimation as opposed to that the conventional Luenberger observer. When the disturbance is not a constant, our approach can constrain the estimation error in a small bounded region and the accuracy of the estimator model increases while the number of past values of the system output used for model fitting is increased.

Remark 2: For a system with external disturbance, the unknown input observer (UIO) design can obtain good state estimation. But UIO has two limits, 1) the relative degree is one; 2) the system must be minimum phase. Based on the second limit, UIO cannot work for observer type output feedback control of the nonminimum phase system. On the other hand, Luenberger observer and the proposed method can estimate the state of the nonminimum phase system. But the estimation performance of Luenberger observer is worst, as mentioned in remrk1. The comparison between Luenberger observer and the proposed method is shown by simulation results.

4. Output tracking controller design

After our having obtaining the estimation system state and disturbance, we proceeded with the design of controller. First, we use the following method of command generator tracker such as Eq. (18) to establish the desired reference model. Then, Lemmas 3 and 4 are used to obtain the associated matrices. Augmenting the integral term of tracking error into system (2), Eq. (27) is acquired. Lemma 5 is used to prove that system (27) is controllable. Finally, the design of controller can be carried out. By using solution obtained from Lyapunov equation (28) in conjunction with a control principle as designed and represented by Eq. (29), the characteristic of our proposed controller is demonstrated by Theorem 2.

The command generator tracker, proposed by Ref. (4), was originally developed for continuous time flight control. The most unique feature of this method is that it has no restrictions of imposing the order of the model to be the same as the plant; thus, the conventional perfect model following constraint can be replaced. As the system dimension is increased, the command generator tracker can be used to save a lot of calculation time. Let the reference output $\boldsymbol{y}_m$ be generated by the following the reference model:

$$\begin{aligned} \boldsymbol{x}_m(k+1) &= (\boldsymbol{A}_m + \boldsymbol{I}_k)\boldsymbol{x}_m(k) + \boldsymbol{B}_m\boldsymbol{u}_m(k) \\ \boldsymbol{y}_m(k) &= \boldsymbol{C}_m\boldsymbol{x}_m(k) \end{aligned} \tag{18}$$

where $\boldsymbol{x}_m \in \mathbb{R}^k$ and $\boldsymbol{u}_m \in \mathbb{R}^l$ are the state of the reference model and reference input, respectively, and $\boldsymbol{y}_m$ having the same dimension as $\boldsymbol{y}$ is the reference output. Note that the order of $\boldsymbol{x}_m$ is allowed to be unequal to the order of system state $\boldsymbol{x}$. Hence, the model (18) can be of any order that is sufficiently large to maintain the command of the plant. For simplicity, we assume here that the command generator uses the constant $\boldsymbol{u}_m$ to generate the desired output $\boldsymbol{y}_m$. It follows that $\boldsymbol{u}_m(k) = \boldsymbol{u}_m(k+1)$.

Lemma 3: Assume that there exist parameter matrices $\boldsymbol{E} \in \mathbb{R}^{n\times k}$, $\boldsymbol{F} \in \mathbb{R}^{n\times l}$, $\boldsymbol{D} \in \mathbb{R}^{m\times k}$ and $\boldsymbol{H} \in \mathbb{R}^{m\times l}$ such that

$$\begin{bmatrix} \boldsymbol{A}-\boldsymbol{I}_n & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{E} & \boldsymbol{F} \\ \boldsymbol{D} & \boldsymbol{H} \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}\boldsymbol{A}_m & \boldsymbol{E}\boldsymbol{B}_m \\ \boldsymbol{C}_m & \boldsymbol{0} \end{bmatrix}. \tag{19}$$

Let $\boldsymbol{e}_y = \boldsymbol{y} - \boldsymbol{y}_m$ and $\boldsymbol{e}_x = \boldsymbol{x} - \boldsymbol{E}\boldsymbol{x}_m - \boldsymbol{F}\boldsymbol{u}_m$. Then we have

$$\begin{aligned} \boldsymbol{e}_x(k+1) &= \boldsymbol{A}\boldsymbol{e}_x(k) + \boldsymbol{B}\left(\boldsymbol{u}(k) - \boldsymbol{D}\boldsymbol{x}_m(k) - \boldsymbol{H}\boldsymbol{u}_m(k)\right) + \boldsymbol{f}(k) \\ \boldsymbol{e}_y(k) &= \boldsymbol{C}\boldsymbol{e}_x(k). \end{aligned} \tag{20}$$

Proof: From Eqs. (2) and (18), the dynamics of $\boldsymbol{e}_x$ is given by

$$\boldsymbol{e}_x(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k) + \boldsymbol{f}(k) - \boldsymbol{E}\boldsymbol{A}_m\boldsymbol{x}_m(k+1) - (\boldsymbol{E}\boldsymbol{B}_m + \boldsymbol{F})\boldsymbol{u}_m(k).$$

Substituting the matrix relation equations (19) into the above equation yields

$$\begin{aligned} \boldsymbol{e}_x(k+1) &= \boldsymbol{A}\boldsymbol{e}_x(k) + \boldsymbol{B}\left(\boldsymbol{u}(k) - \boldsymbol{D}\boldsymbol{x}_m(k) - \boldsymbol{H}\boldsymbol{u}_m(k)\right) + \boldsymbol{f}(k) \\ \boldsymbol{e}_y(k) &= \boldsymbol{C}\boldsymbol{x}(k) - \boldsymbol{C}_m\boldsymbol{x}_m(k) = \boldsymbol{C}\boldsymbol{e}_x(k). \end{aligned}$$

This completes this proof. □

Lemma 4: Consider a Sylvester equation of the form

$$\boldsymbol{X} + \boldsymbol{A}_s \boldsymbol{X} \boldsymbol{B}_s = \boldsymbol{C}_s \tag{21}$$

where $\boldsymbol{X} \in \mathbb{R}^{p\times q}$ is an unknown matrix, and $\boldsymbol{A}_s$, $\boldsymbol{B}_s$, and $\boldsymbol{C}_s$ are constant matrices of appropriate dimensions. Let $\lambda_i(\boldsymbol{A}_s)$ represent the *i*th eigenvalue of $\boldsymbol{A}_s$ and $vec[\boldsymbol{X}]$ denotes a column vector obtained by stacking all column vectors of $\boldsymbol{X}$ together. Equation (21) has a unique solution if and only if $\lambda_i(\boldsymbol{A}_s)\lambda_j(\boldsymbol{B}_s) \neq -1$ for any *i* and *j*; in this case the unique solution is given by

$$vec[\boldsymbol{X}] = \left[\left(\boldsymbol{B}_s^T \otimes \boldsymbol{A}_s\right) + \boldsymbol{I}_{pq}\right]^{-1} vec[\boldsymbol{C}_s] \tag{22}$$

where $\otimes$ represents the Kronecker product. When $\boldsymbol{C}_s = \boldsymbol{0}$, the homogenous equation also has a unique solution $\boldsymbol{X} = \boldsymbol{0}$.

Proof: See Ref. (7). □

Since (A2) holds, we have

$$\begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} \\ \boldsymbol{\Omega}_{21} & \boldsymbol{\Omega}_{22} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} - \boldsymbol{I}_n & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix}^{-1} \tag{23}$$

where $\boldsymbol{\Omega}_{11} \in \mathbb{R}^{n\times n}$, $\boldsymbol{\Omega}_{12} \in \mathbb{R}^{n\times m}$, $\boldsymbol{\Omega}_{21} \in \mathbb{R}^{m\times n}$, and $\boldsymbol{\Omega}_{22} \in \mathbb{R}^{m\times m}$. It follows from Eqs. (19) and (23) that

$$\begin{bmatrix} \boldsymbol{E} & \boldsymbol{F} \\ \boldsymbol{D} & \boldsymbol{H} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} \\ \boldsymbol{\Omega}_{21} & \boldsymbol{\Omega}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{E}\boldsymbol{A}_m & \boldsymbol{E}\boldsymbol{B}_m \\ \boldsymbol{C}_m & \boldsymbol{0} \end{bmatrix}. \tag{24}$$

Given the known matrices $\boldsymbol{A}_m$, $\boldsymbol{C}_m$, $\boldsymbol{\Omega}_{11}$, and $\boldsymbol{\Omega}_{12}$, and applying Lemma 4 to Eq. (24), if $\lambda_i(\boldsymbol{\Omega}_{11})\lambda_j(\boldsymbol{A}_m) \neq 1$ we can obtain the solution for the matrix $\boldsymbol{E}$ as

$$vec[\boldsymbol{E}] = \left[\left(\boldsymbol{A}_m^T \otimes (-\boldsymbol{\Omega}_{11})\right) + \boldsymbol{I}_{nk}\right]^{-1} vec[\boldsymbol{\Omega}_{12}\boldsymbol{C}_m]. \tag{25}$$

Hence solutions for matrices $\boldsymbol{E}$, $\boldsymbol{F}$, $\boldsymbol{D}$, and $\boldsymbol{H}$ can be obtained. Once these parameter matrices for system (20) have been derived, a control law is then designed such that the system output $\boldsymbol{y}(k)$ can approximately track the reference $\boldsymbol{y}_m(k)$.

Introducing the integral vector of the tracking error as

$$\boldsymbol{\eta}(k+1) = \boldsymbol{\eta}(k) + \boldsymbol{e}_y(k) = \boldsymbol{\eta}(k) + \left(\boldsymbol{y}(k) - \boldsymbol{y}_m(k)\right) \tag{26}$$

where $\boldsymbol{\eta} \in \mathbb{R}^m$, and augmenting Eqs. (7) and (26) with system (20) will yield

$$\begin{aligned} \boldsymbol{z}(k+1) &= \boldsymbol{A}_z \boldsymbol{z}(k) + \boldsymbol{B}_z\left(\boldsymbol{u}(k) - \boldsymbol{D}\boldsymbol{x}_m(k) - \boldsymbol{H}\boldsymbol{u}_m(k) + \boldsymbol{d}(k)\right) + O\left(T^2\right) \\ \boldsymbol{e}_y(k) &= \boldsymbol{C}_z \boldsymbol{z}(k) \end{aligned} \tag{27}$$

where $\boldsymbol{z} = \begin{bmatrix} \boldsymbol{e}_x \\ \boldsymbol{\eta} \end{bmatrix} \in \mathbb{R}^{n+m}$, $\boldsymbol{A}_z = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{C} & \boldsymbol{I}_m \end{bmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}$, $\boldsymbol{B}_z = \begin{bmatrix} \boldsymbol{B}^T & \boldsymbol{0} \end{bmatrix}^T \in \mathbb{R}^{(n+m)\times m}$, and $\boldsymbol{C}_z = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} \in \mathbb{R}^{m\times(n+m)}$.

Lemma 5: If Eq. (2) is controllable and (A2) is satisfied, Eq. (27) is controllable.

Proof: It is similar to the proof of lemma 2. □

Since system (27) is controllable, a matrix $\boldsymbol{P} \in \mathbb{R}^{(n+m)\times(n+m)} > 0$ can be obtained by using the following Riccati equation:

$$\boldsymbol{P} + \boldsymbol{A}_z^T \boldsymbol{P}\boldsymbol{B}_z \left(\boldsymbol{B}_z^T \boldsymbol{P}\boldsymbol{B}_z\right)^{-1} \boldsymbol{B}_z^T \boldsymbol{P}\boldsymbol{A}_z - \boldsymbol{A}_z^T \boldsymbol{P}\boldsymbol{A}_z = \boldsymbol{Q} \tag{28}$$

where $\boldsymbol{Q} \in \mathbb{R}^{(n+m)\times(n+m)} > 0$ is a given matrix. Let $\boldsymbol{K} = \begin{bmatrix}\boldsymbol{K}_1 & \boldsymbol{K}_2\end{bmatrix} = \left(\boldsymbol{B}_z^T \boldsymbol{P}\boldsymbol{B}_z\right)^{-1} \boldsymbol{B}_z^T \boldsymbol{P}\boldsymbol{A}_z$ where $\boldsymbol{K}_1 \in \mathbb{R}^{m\times n}$ and $\boldsymbol{K}_2 \in \mathbb{R}^{m\times m}$. Design the control input as

$$\boldsymbol{u}(k) = \boldsymbol{D}\boldsymbol{x}_m(k) + \boldsymbol{H}\boldsymbol{u}_m(k) - \hat{\boldsymbol{d}}(k) - \boldsymbol{K}_1\hat{\boldsymbol{e}}_x(k) - \boldsymbol{K}_2\boldsymbol{\eta}(k) \tag{29}$$

where $\hat{\boldsymbol{e}}_x(k) = \hat{\boldsymbol{x}}(k) - \boldsymbol{E}\boldsymbol{x}_m(k) - \boldsymbol{F}\boldsymbol{u}_m(k)$. Combine Eqs. (27) and (29) with the estimation error dynamics (9) to obtain

$$\begin{bmatrix}\boldsymbol{z}(k+1)\\ \boldsymbol{w}(k+1)\end{bmatrix} = \begin{bmatrix}\boldsymbol{A}_z - \boldsymbol{B}_z\boldsymbol{K} & \boldsymbol{R}\\ \boldsymbol{0} & \bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\end{bmatrix}\begin{bmatrix}\boldsymbol{z}(k)\\ \boldsymbol{w}(k)\end{bmatrix} + O\left(T^2\right) \tag{30}$$

where $\boldsymbol{R} = \begin{bmatrix}\boldsymbol{B}_z\boldsymbol{K}_1 & \boldsymbol{0} & \boldsymbol{B}_z\end{bmatrix}$. Although the estimation state is used in the controller, it follows from Eq. (30) that the separating principle is valid and the stability is guarantied.

Theorem 2: Consider the dynamic system (2) with the estimator model (6) and design the control law (29). If (A1)-(A3) hold and the disturbance is smooth enough, then the tracking error has the following property:

$$\lim_{k\to\infty}\left\|\boldsymbol{e}_x(k)\right\| \le O(T) \quad \text{and} \quad \lim_{k\to\infty}\left\|\boldsymbol{e}_y(k)\right\| \le O(T).$$

Proof: Since two matrices $\boldsymbol{A}_z - \boldsymbol{B}_z\boldsymbol{K}$ and $\bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}$ are stable, similar to the work of Theorem 1, it follows from Eq. (30) that $\lim_{k\to\infty}\left\|\boldsymbol{z}(k)\right\| \le O(T)$. From $\boldsymbol{z} = \begin{bmatrix}\boldsymbol{e}_x^T & \boldsymbol{\eta}^T\end{bmatrix}^T$ and $\boldsymbol{e}_y = \boldsymbol{C}\boldsymbol{e}_x$, we have $\lim_{k\to\infty}\left\|\boldsymbol{e}_x(k)\right\| \le O(T)$ and $\lim_{k\to\infty}\left\|\boldsymbol{e}_y(k)\right\| \le O(T)$. The proof is complete. □

Remark 3: All eigenvalues of the closed-loop system matrix $\begin{bmatrix}\boldsymbol{A}_z - \boldsymbol{B}_z\boldsymbol{K} & \boldsymbol{R}\\ \boldsymbol{0} & \bar{\boldsymbol{A}} - \boldsymbol{L}\bar{\boldsymbol{C}}\end{bmatrix}$ are still in the stable region even the system is nonminimum phase. The last term in Eq. (30) does not affect the stability of the closed-loop system. Using the command generator tracker technology, our algorithm can accomplish the output tracking problem even if system (2) is nonminimum phase. Figure 1 shows the total controller block diagram.

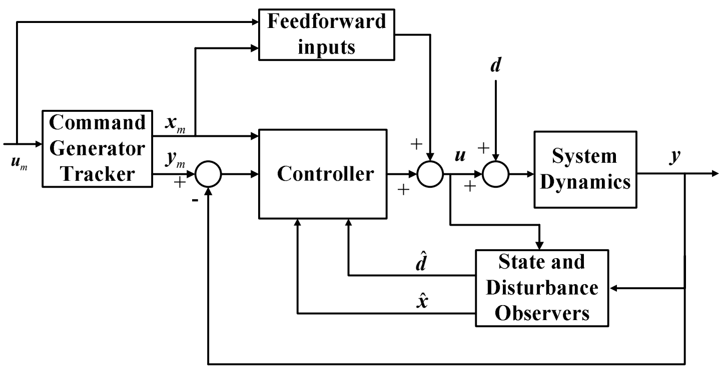

Fig. 1. Total control block diagram.

5. Numerical example

A third-order DC motor system[14] is proposed to illustrate the control algorithm where position, velocity and armature current are taken as the state and only the angular position measurement has been used. The system matrices are shown as follows,

$$\boldsymbol{A}=\begin{bmatrix}1 & 0.0646 & 0.0065\\ 0 & 0.6282 & 0.0791\\ 0 & -0.3591 & -0.0430\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}0.0405\\ 1.1060\\ 1.5651\end{bmatrix},\ \boldsymbol{C}=\begin{bmatrix}1 & 0 & 0\end{bmatrix},\ d(k)=0.2\sin(0.03k)+1.$$

The triple $(\boldsymbol{C},\boldsymbol{A},\boldsymbol{B})$ itself has an unstable transmission zeros at -1.3919, hence; it is nonminimum phase. Design the observer (6) where $\boldsymbol{F}=\begin{bmatrix}0.1852\\ 21.9274\\ -162.0166\end{bmatrix}$, $l_1=0.0469$, and $l_2=0.5774$. For comparing the estimation performances between the proposed method and Luenberger observer, the matrix $\boldsymbol{L}$ for Luenberger observer is designed as $\boldsymbol{L}=\boldsymbol{F}$. The reference model is given by $\boldsymbol{x}_m(k+1)=\begin{bmatrix}0 & 1\\ -1 & 1.9996\end{bmatrix}\boldsymbol{x}_m(k)$, $y_m(k)=\begin{bmatrix}0 & 1\end{bmatrix}\boldsymbol{x}_m(k)$, and the parameter matrices $\boldsymbol{E}$ and $\boldsymbol{D}$ can be directly calculated by $\boldsymbol{E}=\begin{bmatrix}0 & 1\\ -12.7007 & 12.7007\\ -1.8369 & 1.8369\end{bmatrix}$ and $\boldsymbol{D}=\begin{bmatrix}-4.1382 & 4.1382\end{bmatrix}$. Setting $\boldsymbol{Q}=\boldsymbol{I}_4$ and solving the Riccati equation (28) can obtain the control law as

$$\eta(k+1)=\eta(k)+\left(y(k)-y_m(k)\right)$$
$$u(k)=-\begin{bmatrix}2.7706 & 0.3502 & 0.0407\end{bmatrix}\left(\hat{\boldsymbol{x}}(k)-\boldsymbol{E}\boldsymbol{x}_m(k)\right)-0.3751\eta(k)-\hat{d}(k)+\boldsymbol{D}\boldsymbol{x}_m(k).$$

The eigenvalues of the closed-loop system (30) are all stable $\{0, 0.4105, 0.5983, 0.8057\pm j0.1620, 0.6494\pm j0.2004, 0.7515\pm j0.1012\}$.

Under initial state $\boldsymbol{x}(0)=\begin{bmatrix}0.1 & 0 & 0\end{bmatrix}^T$ and $\hat{\boldsymbol{x}}(0)=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$, the comparison between the proposed method and Luenberger observer is depicted in Fig. 2. We can find that both two observers can be converged and the estimation error $\|\tilde{\boldsymbol{x}}\|$ for the proposed observer is smaller than Luenberger's. The estimation performances of the system state and the disturbance are shown in Figs. 3 and 4, respectively. Obviously, when $k>50$, the system state and the unknown disturbance can be estimated with great accuracy according to the figures shown in this paper. Therefore, in spite of the fact that the system is nonminimum phase, the estimation method we propose can be functioning normally to obtain valid estimation. Figure 5 shows trajectories of the system output $y(k)$ and the reference $y_m(k)$, and the input response is in Fig. 6. Based on these figures, although the system is in the presence of disturbance and has an unstable zero, the system performance is excellent using the proposed algorithm.

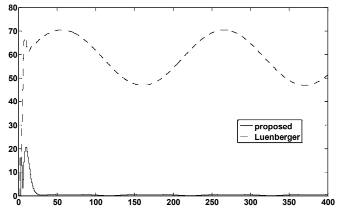

Fig. 2. $\|\tilde{x}\|$ of proposed method and Luenberger observer.

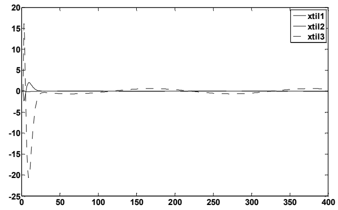

Fig. 3. Estimation errors of the system state.

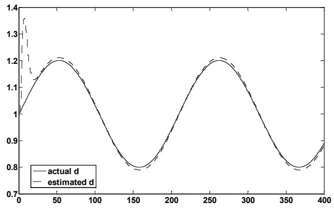

Fig. 4. Estimation performance of d.

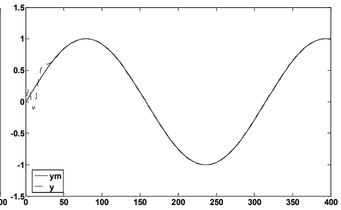

Fig. 5. The trajectories of y and y_m.

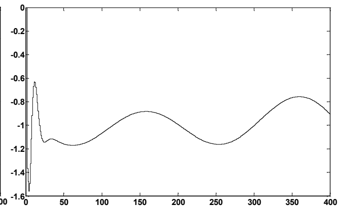

Fig. 6. Control input.

6. Conclusion

In this paper, based upon the fashion of state and disturbance observers, an algorithm developed in with a discrete time nonminimum phase MIMO system to implement output tracking controller design is proposed. First, a new estimator structure is proposed to estimate both system state and unknown disturbance. Admitting that a perfect estimation performance is unachievable, the algorithm proposed here can confine the estimation error to be smaller than $O(T)$. In addition, estimation signals are used to be embedded into the design of controller of which it contains proportional-integral feedback and feedforward input. Further, the control method is proven to be capable of containing the tracking error within the range of $O(T)$. Using these estimation signals, a control law combining the proportional-integral feedback control and the feedforward input is then devised. In our algorithm, the output can approximately track the reference with the small tracking error being bounded. Simulation results show that the present scheme exhibits reasonably good performance when the underlying system has an unstable zero.

Acknowledgment

This work was supported by the National Science Council, Taiwan, R.O.C., under Contract NSC 96-2221-E-161-005.

References

(1) Abidi, K., Xu, J.X., and Xinghuo, Y., On the Discrete-Time Integral Sliding Mode Control, *IEEE Transactions on Automatic Control*, Vol.52, No.4, 2007, pp.709-715.

(2) Astron, K.J. and Wittenmark, B., Computer Controlled Systems-Theory and Design, Second Edition, 1990, Prentice-Hall, Englewood Cliffs, NJ.

(3) Beale, S. and Shafai, B., Robust Control Design with a Proportional Integral Observer, *International Journal of Control*, Vol.50, 1989, pp.97-111.

(4) Broussard, J. and O'Brien, M., Feedforward Control to Track the Output of a Forced Model, *IEEE Transactions on Automatic Control*, Vol.25, No.4, 1980, pp.851-853.

(5) Busawon, K.K. and Kabore, P., Disturbance Attenuation Using Proportional Integral Observers, *International Journal of Control*, Vol.74, No.6, 2001, pp.618-627.

(6) Darouach, M., Zasadzinski, M., and Xu, S.J., Full-Order Observers for Linear systems, *IEEE Transactions on Automatic Control*, Vol.39, No.3, 1994, pp.606-609.

(7) Ding, F. and Chen, T., Gradient Based Iterative Algorithms for Solving a Class of Matrix Equation, *IEEE Transactions on Automatic Control*, Vol.50, No.8, 2005, pp.1216-1221.

(8) Hacck, B. and Tomizuka, M., The Effect of Adding Zeros to Feedforward Controller, *ASME Journal of Dynamics System, Measurement and Control*, Vol.113, No.1, 1991, pp.6-10.

(9) Hori, Y., Robust Motion Control Based on a Two-Degree-of-Freedom Servo System, *Advanced Robotics*, Vol.7, 1993, pp.363-368.

(10) Hou, M. and Muller, P.C., Design of Observers for Linear Systems with Unknown Inputs, *IEEE Transactions on Automatic Control*, Vol.37, No.6, 1992, pp.871-875.

(11) Gross, E. and Tomizuka, M., Experimental Flexible Beam Tip Tracking Control with a Truncated Series Approximation to Uncancelable Inverse Dynamics, *IEEE Transaction on Control System Technology*, Vol.2, No.4, 1994, pp.382-391.

(12) Jemaa, L.B. and Davison, E., Performance Limitations in the Robust Servomechanism Problem for Discrete-Time LTI Systems, *IEEE Transactions on Automatic Control*, Vol.48, No.8, 2003, pp.1299-1311.

(13) Kempf, C.J. and Kobayashi, S., Disturbance Observer and Feedforward Design for a High-Speed Direct Drive Position Table, *IEEE Transaction on Control System Technology*, Vol.7, No.5, 1999, pp.513-526.

(14) Lai, N.O., Edwards, C., and Spurgeon, S.K., Discrete Output Feedback Sliding-Mode Control with Integral Action, *International Journal of Robust and Nonlinear Control*, Vol.16, No.1, 2006, pp.21-43.

(15) Marconi, L., Marro, G., and Melchiorri, C., A Solution Technique for Almost Perfect Tracking of Non-Minimum-Phase Discrete-Time Linear Systems, *International Journal of Control*, Vol.74, No.5, 2001, pp.496-506.

(16) Shafai, B., Beale, S., Niemann, H.H., and Stoustrup, J.L., LTR Design for Discrete-Time Proportional-Integral Observers, *IEEE Transactions on Automatic Control*, Vol.41, No.7, 1996, pp.1056-1062.

(17) Su, W.C., Drakunov, S.V., and Ozguner, U., An O(T^2) Boundary Layer in Sliding Mode for Sampled–Data Systems, *IEEE Transactions on Automatic Control*, Vol.45, No.3, 2000, pp.482-485.

(18) Tomizuka, M., On the Design of Digital Tracking Controllers, *ASME Journal of Dynamics System, Measurement and Control*, Vol.115, No.2, 1993, pp.412-418.

(19) Tomizuka, M., Zero Phase Error Tracking Algorithm for Digital Control, *ASME Journal of Dynamics System, Measurement and Control*, Vol.109, 1987, pp.65-68.

(20) Xia, J.K. and Menq, C.H., Precision Tracking Control of Nonminimum-Phase Systems with Zero Phase Error, *International Journal of Control*, Vol.61, 1995, pp.791-807.