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OUTPUT FEEDBACK INTEGRAL SLIDING MODE CONTROLLER OF TIME-DELAY SYSTEMS WITH MISMATCH DISTURBANCES

Huan-Chan Ting, Jeang-Lin Chang, and Yon-Ping Chen

ABSTRACT

For uncertain time-delay systems with mismatch disturbances, this paper presented an integral sliding mode control algorithm using output information only. An integral sliding surface is comprised of output signals and an auxiliary full-order compensator. The designed output feedback sliding mode controller can locally satisfy the reaching and sliding condition and maintain the system on the sliding surface from the initial moment. Since the system is in the sliding mode and two specific algebraic Riccati inequalities are established, the proposed algorithm can guarantee the stability of the closed-loop system and satisfy the property of disturbance attenuation. Moreover, the design parameters of the controller and compensator can be simultaneously determined by solutions to two algebraic Riccati inequalities. Finally, a numerical example illustrates the applicability of the proposed scheme.

Key Words: Output feedback, full-order compensator, sliding mode, mismatch disturbance.

I. INTRODUCTION

The time-delay phenomenon means that parts of system states, inputs, or outputs affect the system after a fixed time, or random but finite period. This phenomenon exists in various practical systems, such as chemical processes, electrical networks, nuclear reactors, biological systems, and economic models. Since time delays frequently induce system instability and bad performance, the analysis and control of time-delay systems, whether state, input, or output delay, have been an interesting topic over the past decades. Focusing on state delay systems, researchers [1–6] have presented many effective state feedback control methods for various system models. Xia and Jia [1] developed an excellent control method comprising of sliding mode and linear matrix inequality (LMI) technique for uncertain time-delay systems with matched disturbances. Lee *et al*. [6] developed a control method based on the receding horizon concept to stabilize a closed-loop system and to satisfy the H_∞ norm bound from disturbances to the controlled outputs.

When the system states cannot be obtained completely, state observers [7–11] and output feedback control methods [12–17] are both feasible designed schemes. In the field of state observers, Darouach [11] have recently developed an observer methodology to estimate states of linear time-delay systems with noises and mismatch disturbances. On the other hand, in the field of output feedback control methods, Fridman and Shaked [12] described explicitly a significant H_∞ control method using the descriptor system transformation for time-delay systems with mismatch external disturbances and measurement noises. The descriptor system transformation can simplify the analysis of time-delay systems and have effective disturbance attenuation. Niu *et al.* [13] extended an observer-type sliding mode control using LMI technique to regulate

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Huan-Chan Ting (corresponding author) and Yon-Ping Chen are with Department of Electrical and Control Engineering, National Chiao Tung University, Taiwan (e-mail: hcting.ece92g@nctu.edu.tw).

Jeang-Lin Chang is with Department of Electrical Engineering, Oriental Institute of Technology, Pan-Chiao City, Taipei County, Taiwan.

uncertain time-delay systems. Yan *et al*. [16] applied an effective sliding mode design technique using output only to control the time-delay systems with disturbances.

There exist two difficulties in the design of output feedback sliding mode control. The first difficulty is the synthesis problem. Synthesizing a control law using the outputs only is significant since the derivative of the sliding surface is always involved with the unmeasured system states. For determining the feedback gain to stabilize the closed-loop system, the other difficulty is to solve an LMI, which is necessary and complicated with large dimensions. In this paper, an output feedback integral sliding mode controller combining with a full-order compensator is proposed to improve these two difficulties for uncertain time-delay systems with mismatch disturbances. Since mismatch disturbances cannot be eliminated completely once a system is in the sliding mode, a disturbance attenuation technique [18–20] can reduce the effect of the disturbance acting on a system to an acceptable level. The H_∞ robust control method for disturbance attenuation is a successful strategy to minimize the gain from external disturbances to the controlled output over all frequencies. Introducing H_∞ control technology into the proposed method, we can guarantee the stability of the closed-loop system and accomplish the property of robust disturbance attenuation. Moreover, the proposed method adopts two algebraic Riccati inequalities to determine the parameters in the integral sliding surface.

This paper is organized as follows. The description of time-delay systems and the problem formulation are given in Section II. Section III is divided into three parts. In subsection 3.1, the integral sliding surface is designed and the output feedback sliding mode control law is used to satisfy the reaching and sliding condition. The designed full-order compensator and the closed-loop system are given in subsection 3.2. Subsection 3.3 shows that the solutions to two algebraic Riccati inequalities guarantee the robust stability once a system is in the sliding mode. The feasibility of the proposed method is illustrated in Section IV with a numerical example. Conclusions are given in Section V.

II. PROBLEM FORMULATION

Consider a continuous-time time-delay system described by the state-space form as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{A}_d\boldsymbol{x}(t-\tau)+\boldsymbol{B}(\boldsymbol{u}(t)\\ &\quad +\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t))+\boldsymbol{E}\boldsymbol{d}(t)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)\\ \boldsymbol{x}(t) &= \phi(t), \quad t\in[-\tau,0]\end{aligned} \tag{1}$$

where $\boldsymbol{x}\in\mathbb{R}^n$ is the system state vector, $\boldsymbol{y}\in\mathbb{R}^l$ is the system output vector, $\boldsymbol{u}\in\mathbb{R}^m$ is the control input vector, $\boldsymbol{d}\in\mathbb{R}^p$ is the mismatched disturbance vector. The function $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t)\in\mathbb{R}^m$ is unknown and represents the matched uncertainty. The constant τ is an unknown delay time but bounded by a known constant τ^*, where $\tau\le\tau^*$. The vector $\phi(t)$ is a continuous initial function. The real constant matrices $\boldsymbol{A}, \boldsymbol{A}_d, \boldsymbol{B}, \boldsymbol{E}$, and $\boldsymbol{C}$ are known and have appropriate dimensions with $l\ge m$. The term $\boldsymbol{A}_d\in\mathbb{R}^{n\times n}$ is a known and real constant matrix. Notice that $\boldsymbol{A}_d$ can be mismatched, i.e. $\boldsymbol{A}_d\ne\boldsymbol{B}\Xi$, where Ξ is an arbitrary matrix with an appropriate dimension. Suppose that system (1) is completely controllable and observable. Spurgeon and Edwards [21] have shown that there exists a stable static output feedback sliding mode controller if

(C1) $\operatorname{rank}(\boldsymbol{CB})=\operatorname{rank}(\boldsymbol{B})=m$,

(C2) The triple $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C})$ is minimum phase.

Moreover, the method of ensuring robust disturbance attenuation is to design a controller such that the closed-loop system is stable and a constant $0\le\bar{\gamma}<\infty$ exists for which the performance bound [18, 19]

$$\int_0^t(\boldsymbol{y}^T\boldsymbol{y}+\boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u})dq\le\bar{\gamma}^2\int_0^t(\boldsymbol{d}^T\boldsymbol{d})dq \quad \forall t\ge 0 \tag{2}$$

is satisfied, where $\boldsymbol{R}>0$ is the input weighting matrix. To focus on time-delay systems satisfying conditions (C1) and (C2), we design the integral sliding surface applying the full-order compensator in which the proposed control algorithm can guarantee the performance bound (2) of robust disturbance attenuation once the system is in the sliding mode. The control law involved the output information and the compensator is designed to satisfy the reaching and sliding condition. Before introducing this control law, the following three assumptions are fulfilled throughout this paper.

Assumption II.1. The vector $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t)$ and mismatch disturbance $\boldsymbol{d}(t)$ are norm-bounded as

$$\|\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},t)\|\le\rho(t,\boldsymbol{y})+\chi\|\boldsymbol{u}(t)\| \quad \text{and} \quad \|\boldsymbol{d}(t)\|\le\bar{d}$$

where $0<\chi<1$, $\rho(t,\boldsymbol{y})$, and $\bar{d}$ are known positive constants and $\|\bullet\|$ denotes the 2-norm.

Assumption II.2. The triple $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C})$ is minimum phase.

Assumption II.3. $\operatorname{rank}(\boldsymbol{CB})=\operatorname{rank}(\boldsymbol{B})=m$.

III. INTEGRAL SLIDING MODE CONTROLLER DESIGN

In this section, the output feedback controller using the integral sliding surface is proposed by employing the full-order compensator. Once the system is in the sliding mode, the proposed algorithm can guarantee the stability of the closed-loop system and sustain the nature of disturbance attenuation when two algebraic Riccati inequalities can be solved.

3.1 Sliding surface and sliding mode controller

Since Assumption II.3 holds, we design the output-dependant integral sliding surface as

$$\boldsymbol{s}(t) = (\boldsymbol{GCB})^{-1}\boldsymbol{G}(\boldsymbol{y}(t) - \boldsymbol{y}(0)) - \int_0^t \boldsymbol{v}(q)dq \tag{3}$$

where $\boldsymbol{G} \in \mathbb{R}^{m\times l}$ is chosen such that $\boldsymbol{GCB}$ is invertible and $\boldsymbol{v} \in \mathbb{R}^m$ is designed in the latter. Castanos and Fridman [22] mentioned the state-dependent integral sliding surface design for linear systems to ensure robust disturbance attenuation. Differentiating (3) with respect to time and substituting system (1) into it can obtain

$$\begin{aligned}\dot{\boldsymbol{s}}(t) &= (\boldsymbol{GCB})^{-1}\boldsymbol{GC}[\boldsymbol{Ax}(t) + \boldsymbol{A}_d\boldsymbol{x}(t-\tau) \\ &\quad + \boldsymbol{B}(\boldsymbol{u}(t) + \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)) + \boldsymbol{Ed}(t)] - \boldsymbol{v}(t) \\ &= (\boldsymbol{GCB})^{-1}\boldsymbol{GCAx}(t) + \boldsymbol{u}(t) + \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \\ &\quad - \boldsymbol{v}(t) + (\boldsymbol{GCB})^{-1}\boldsymbol{GC}(\boldsymbol{A}_d\boldsymbol{x}(t-\tau) + \boldsymbol{Ed}(t)).\end{aligned} \tag{4}$$

Define two regions Ω_1 and Ω_2 as [23]

$$\begin{aligned}\Omega_1 &:= \{\boldsymbol{x}(t) \mid \|(\boldsymbol{GCB})^{-1}\boldsymbol{GCAx}\| \le \sigma_1\} \subset \Omega \\ \Omega_2 &:= \{\boldsymbol{x}(t-\tau) \mid \|(\boldsymbol{GCB})^{-1}\boldsymbol{GCA}_d\boldsymbol{x}(t-\tau)\| \le \sigma_2\} \subset \Omega\end{aligned} \tag{5}$$

where $\sigma_1 > 0$, $\sigma_2 > 0$, and the region $\Omega \subset \mathbb{R}^n$ is a neighborhood of the origin. Consider system (1) in $\Omega_1 \times \Omega_2$ and design the control input as

$$\boldsymbol{u}(t) = \boldsymbol{v}(t) - \kappa(t)\frac{\boldsymbol{s}(t)}{\|\boldsymbol{s}(t)\|} \tag{6}$$

where $\kappa(t) = \frac{1}{1-\chi}(\sigma_1 + \sigma_2 + \rho(t, y) + \chi\|\boldsymbol{v}(t)\| + \psi\bar{d} + \mu)$ and the positive constants σ_1 and σ_2 are given in (5). The control parameters $\psi = \|(\boldsymbol{GCB})^{-1}\boldsymbol{GCE}\|$ and μ are positive constants. Through straightforward calculation, we have

$$\begin{aligned}\kappa(t) &= \chi\kappa(t) + \chi\|\boldsymbol{v}(t)\| + \sigma_1 + \sigma_2 + \rho(t, y) + \psi\bar{d} + \mu \\ &\ge \chi\|\boldsymbol{u}(t)\| + \sigma_1 + \sigma_2 + \rho(t, y) + \psi\bar{d} + \mu.\end{aligned}$$

Substituting (6) into (4) and then pre-multiplying $\boldsymbol{s}^T$ in both sides of (4) can attain

$$\begin{aligned}\boldsymbol{s}^T(t)\dot{\boldsymbol{s}}(t) &= \boldsymbol{s}^T(t)((\boldsymbol{GCB})^{-1}\boldsymbol{GCAx}(t) \\ &\quad + \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) + (\boldsymbol{GCB})^{-1}\boldsymbol{GC}(\boldsymbol{A}_d\boldsymbol{x}(t-\tau) \\ &\quad + \boldsymbol{Ed}(t))) - \kappa(t)\|\boldsymbol{s}(t)\| \\ &\le ((\boldsymbol{GCB})^{-1}\boldsymbol{GCAx}(t) + \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \\ &\quad + (\boldsymbol{GCB})^{-1}\boldsymbol{GC}(\boldsymbol{A}_d\boldsymbol{x}(t-\tau) \\ &\quad + \boldsymbol{Ed}(t)) - \kappa(t))\|\boldsymbol{s}(t)\| \\ &\le (\alpha\|\boldsymbol{x}(t)\| + \beta\|\boldsymbol{x}(t-\tau)\| + \rho(t, y) \\ &\quad + \chi\|\boldsymbol{u}(t)\| + \psi\bar{d} - \kappa(t))\|\boldsymbol{s}(t)\| \\ &\le (\alpha\|\boldsymbol{x}(t)\| + \beta\|\boldsymbol{x}(t-\tau)\| - \sigma_1 \\ &\quad - \sigma_2 - \mu)\|\boldsymbol{s}(t)\| \\ &\le -\mu\|\boldsymbol{s}\|\end{aligned}$$

where $\alpha = \|(\boldsymbol{GCB})^{-1}\boldsymbol{GCA}\|$ and $\beta = \|(\boldsymbol{GCB})^{-1}\boldsymbol{GCA}_d\|$, and the reachability condition is satisfied. Since $\boldsymbol{s}(0) = \boldsymbol{0}$, the controller (6) can guarantee the following identities:

$$\boldsymbol{s}(t) = \dot{\boldsymbol{s}}(t) = \boldsymbol{0} \quad \forall t \ge 0.$$

Therefore, the transient time to the sliding mode can be shortened efficiently. Subsequently, we focus on the stability analysis when the system is in the sliding mode.

From (4), once the system is in the sliding mode, $\boldsymbol{s}(t) = \boldsymbol{0}$, the corresponding equivalent control [21] is given by

$$\begin{aligned}\boldsymbol{u}_{eq}(t) + \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}_{eq}, t) &= -(\boldsymbol{GCB})^{-1}\boldsymbol{GC}[\boldsymbol{Ax}(t) \\ &\quad + \boldsymbol{A}_d\boldsymbol{x}(t-\tau) + \boldsymbol{Ed}(t)] + \boldsymbol{v}(t).\end{aligned} \tag{7}$$

Substituting (7) into system (1) obtains the closed-loop system dynamics in the sliding mode as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{Ax}(t) + \boldsymbol{A}_d\boldsymbol{x}(t-\tau) + \boldsymbol{Bf}(\boldsymbol{x}, \boldsymbol{u}_{eq}, t) \\ &\quad + \boldsymbol{Ed}(t) - \boldsymbol{Bf}(\boldsymbol{x}, \boldsymbol{u}_{eq}, t) + \boldsymbol{Bv}(t) \\ &\quad - \boldsymbol{B}(\boldsymbol{GCB})^{-1}\boldsymbol{GC}[\boldsymbol{Ax}(t) + \boldsymbol{A}_d\boldsymbol{x}(t-\tau) + \boldsymbol{Ed}(t)] \\ &= (\boldsymbol{I} - \boldsymbol{B}(\boldsymbol{GCB})^{-1}\boldsymbol{GC})[\boldsymbol{Ax}(t) + \boldsymbol{A}_d\boldsymbol{x}(t-\tau) \\ &\quad + \boldsymbol{Ed}(t)] + \boldsymbol{Bv}(t) \\ &= \bar{\boldsymbol{A}}\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_d\boldsymbol{x}(t-\tau) + \bar{\boldsymbol{E}}\boldsymbol{d}(t) + \boldsymbol{Bv}(t)\end{aligned} \tag{8}$$

where

$$\bar{A} = (I_n - B(GCB)^{-1}GC)A,$$
$$\bar{A}_d = (I_n - B(GCB)^{-1}GC)A_d, \quad \text{and}$$
$$\bar{E} = (I_n - B(GCB)^{-1}GC)E.$$

Since system (1) is controllable, we can easily show that the pair $(\bar{A}, B)$ is controllable. The present objective is to design the control input v which can assure the property of robust disturbance attenuation (2) when the system is in the sliding mode. For the closed-loop system (8), robust disturbance attenuation is changed to

$$\int_0^t (y^T y + v^T R v)dq \le \gamma^2 \int_0^t (d^T d)dq. \tag{9}$$

Remark III.1. In [22], the integration term in the sliding manifold can be thought as a trajectory of the system in the absence of perturbations and in the presence of the nominal control, that is, as a nominal trajectory for a given initial condition. In this paper, adding the integration term $v(t)$ into the sliding surface (3) can compensate the degree of freedom to attenuate the effects of disturbances and uncertainties in the closed-loop system. Involving the integrator is also helpful to analyze the stability and robustness of the closed-loop system.

3.2 Full-order compensator

Since Assumption II.3 holds, we design a matrix $U = -B(CB)^+ + Y(I_l - CB(CB)^+)$ where $(CB)^+ = ((CB)^T CB)^{-1}(CB)^T$, and $Y \in \mathbb{R}^{n\times l}$ is an arbitrary matrix. Design matrices $M = I_n + UC$ and $H = L(I_l + CU) - MAU$, where L is a gain matrix designed in the latter. According to Assumption II.3, we have $MB = 0$ and $\text{rank}(M) = n - m$. Then the input vector v is generated from the following full-order dynamic compensator:

$$\begin{aligned} \dot{\xi}(t) &= (MA - LC + F)\xi(t) + (H - FU)y(t) \\ v(t) &= -K(\xi(t) - Uy(t)) \end{aligned} \tag{10}$$

where $\xi \in \mathbb{R}^n$ is the auxiliary state. Moreover, $K \in \mathbb{R}^{m\times n}$ and $F \in \mathbb{R}^{n\times n}$ are gain matrices decided in the latter. Define an error vector $e = Mx - \xi$. According to (10) and $MB = 0$, the dynamics of e can be given by

$$\begin{aligned} \dot{e}(t) &= MAx(t) + MA_d x(t-\tau) + MEd(t) \\ &\quad - (MA - LC + F)\xi(t) - (H - FU)y(t) \\ &= (MA - LC)Mx(t) + MA_d x(t-\tau) \\ &\quad + MEd(t) - (MA - LC)\xi(t) - Fx(t) + Fe(t) \\ &= (MA - LC + F)e(t) - Fx(t) + MA_d x(t-\tau) \\ &\quad + MEd(t). \end{aligned} \tag{11}$$

On the other hand, from (10) we rewrite the vector v as

$$v(t) = -Kx(t) + Ke(t). \tag{12}$$

Substitute (12) into (8) to obtain the dynamic equation of x in the sliding mode as

$$\begin{aligned} \dot{x}(t) &= (\bar{A} - BK)x(t) + BKe(t) \\ &\quad + \bar{A}_d x(t-\tau) + \bar{E}d(t). \end{aligned} \tag{13}$$

Combining (11) with (13) can obtain the following closed-loop system:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} \bar{A} - BK & BK \\ -F & MA - LC + F \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} \bar{A}_d & 0 \\ MA_d & 0 \end{bmatrix} \begin{bmatrix} x(t-\tau) \\ e(t-\tau) \end{bmatrix} + \begin{bmatrix} \bar{E} \\ ME \end{bmatrix} d(t), \tag{14}$$

To satisfy the disturbance attenuation property (9), we define the controlled output $z \in \mathbb{R}^{l+m}$ as

$$z(t) = \begin{bmatrix} C \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ D \end{bmatrix} v(t) = \begin{bmatrix} C \\ -DK \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ DK \end{bmatrix} e(t) = \bar{C}x(t) + \bar{D}e(t) \tag{15}$$

where $D^T D = R$.

3.3 Robust disturbance attenuation

Referring to [18], we first define a quadratic energy function as

$$E(x, e) = x^T P_x x + e^T P_e e + \int_{t-\tau}^t x^T(\alpha) Q_x x(\alpha)\, d\alpha + \int_{t-\tau}^t e^T(\alpha) Q_e e(\alpha)\, d\alpha \tag{16}$$

where the matrices $P_x > 0$, $P_e > 0$, $Q_x > 0$, and $Q_e > 0$ are determined in a latter. Define the Hamiltonian function

$$H[d] = z^T z - \gamma^2 d^T d + \frac{dE}{dt} \tag{17}$$

where $\frac{dE}{dt}$ is the derivative of E along the trajectory of the closed-loop system. A sufficient condition for achieving robust disturbance attenuation (9) is that

$$H[d] < 0, \quad \text{for all } d \in L^2 \tag{18}$$

Under (18), $E(\boldsymbol{x},\boldsymbol{e})$ is a strict radially unbounded Lyapunov function of the closed-loop system, and hence the robust stability is guaranteed. We establish conditions under which

$$\sup_{d\in L^2} H[\boldsymbol{d}]<0.$$

From the expression of (16), we can derive

$$\begin{aligned}\frac{dE}{dt} &= ((\bar{\boldsymbol{A}}-\boldsymbol{BK})\boldsymbol{x}(t)+\boldsymbol{BKe}(t)+\bar{\boldsymbol{E}}\boldsymbol{d}(t)\\ &\quad +\bar{\boldsymbol{A}}_d\boldsymbol{x}(t-\tau))^T\boldsymbol{P}_x\boldsymbol{x}(t)\\ &\quad +\boldsymbol{x}^T(t)\boldsymbol{P}_x((\bar{\boldsymbol{A}}-\boldsymbol{BK})\boldsymbol{x}(t)+\boldsymbol{BKe}(t)\\ &\quad +\bar{\boldsymbol{E}}\boldsymbol{d}(t)+\bar{\boldsymbol{A}}_d\boldsymbol{x}(t-\tau))\\ &\quad +(-\boldsymbol{Fx}(t)+(\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})\boldsymbol{e}(t)\\ &\quad +\boldsymbol{MEd}(t)+\boldsymbol{MA}_d\boldsymbol{x}(t-\tau))^T\boldsymbol{P}_e\boldsymbol{e}(t)\\ &\quad +\boldsymbol{e}^T(t)\boldsymbol{P}_e(-\boldsymbol{Fx}(t)+(\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})\boldsymbol{e}(t)\\ &\quad +\boldsymbol{MEd}(t)+\boldsymbol{MA}_d\boldsymbol{x}(t-\tau))\\ &\quad +\boldsymbol{x}^T(t)\boldsymbol{Q}_x\boldsymbol{x}(t)-\boldsymbol{x}^T(t-\tau)\boldsymbol{Q}_x\boldsymbol{x}(t-\tau)\\ &\quad +\boldsymbol{e}^T(t)\boldsymbol{Q}_e\boldsymbol{e}(t)-\boldsymbol{e}^T(t-\tau)\boldsymbol{Q}_e\boldsymbol{e}(t-\tau).\end{aligned}$$

Noting that $z(t)=\bar{\boldsymbol{C}}\boldsymbol{x}(t)+\bar{\boldsymbol{D}}\boldsymbol{e}(t)$, (17) can be rewritten as

$$\begin{aligned}H[\boldsymbol{d}] &= z^Tz-\gamma^2\boldsymbol{d}^T\boldsymbol{d}+\frac{dE}{dt}\\ &= \boldsymbol{x}^T(t)(\boldsymbol{C}^T\boldsymbol{C}+\boldsymbol{K}^T\boldsymbol{RK})\boldsymbol{x}(t)\\ &\quad -\boldsymbol{e}^T(t)\boldsymbol{K}^T\boldsymbol{RKx}(t)-\boldsymbol{x}^T(t)\boldsymbol{K}^T\boldsymbol{RKe}(t)\\ &\quad +\boldsymbol{e}^T(t)\boldsymbol{K}^T\boldsymbol{RKe}(t)+((\bar{\boldsymbol{A}}-\boldsymbol{BK})\boldsymbol{x}(t)\\ &\quad +\boldsymbol{BKe}(t)+\bar{\boldsymbol{A}}_d\boldsymbol{x}(t-\tau))^T\boldsymbol{P}_x\boldsymbol{x}(t)\\ &\quad +\boldsymbol{x}^T(t)\boldsymbol{P}_x((\bar{\boldsymbol{A}}-\boldsymbol{BK})\boldsymbol{x}(t)+\boldsymbol{BKe}(t)+\bar{\boldsymbol{A}}_d\boldsymbol{x}(t-\tau))\\ &\quad +(-\boldsymbol{Fx}(t)+(\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})\boldsymbol{e}(t)\\ &\quad +\boldsymbol{MA}_d\boldsymbol{x}(t-\tau))^T\boldsymbol{P}_e\boldsymbol{e}(t)+\boldsymbol{e}^T(t)\boldsymbol{P}_e(-\boldsymbol{Fx}(t)\\ &\quad +(\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})\boldsymbol{e}(t)+\boldsymbol{MA}_d\boldsymbol{x}(t-\tau))\\ &\quad +\boldsymbol{x}^T(t)\boldsymbol{Q}_x\boldsymbol{x}(t)-\boldsymbol{x}^T(t-\tau)\boldsymbol{Q}_x\boldsymbol{x}(t-\tau)\\ &\quad +\boldsymbol{e}^T(t)\boldsymbol{Q}_e\boldsymbol{e}(t)-\boldsymbol{e}^T(t-\tau)\boldsymbol{Q}_e\boldsymbol{e}(t-\tau)\\ &\quad -\gamma^2\boldsymbol{d}^T(t)\boldsymbol{d}(t)+\boldsymbol{d}^T(t)\bar{\boldsymbol{E}}^T\boldsymbol{P}_x\boldsymbol{x}(t)\\ &\quad +\boldsymbol{d}^T(t)\boldsymbol{E}^T\boldsymbol{M}^T\boldsymbol{P}_e\boldsymbol{e}(t)+\boldsymbol{x}^T(t)\boldsymbol{P}_x\bar{\boldsymbol{E}}\boldsymbol{d}(t)\\ &\quad +\boldsymbol{e}^T(t)\boldsymbol{P}_e\boldsymbol{MEd}(t).\end{aligned}$$

Based on the above equation, the worst case $\sup_{d\in L^2} H[\boldsymbol{d}]$ occurs when

$$\boldsymbol{d}(t)=\gamma^{-2}(\bar{\boldsymbol{E}}^T\boldsymbol{P}_x\boldsymbol{x}(t)+\boldsymbol{E}^T\boldsymbol{M}^T\boldsymbol{P}_e\boldsymbol{e}(t))$$

and it implies that

$$\begin{aligned}H[\boldsymbol{d}] &\le \boldsymbol{x}^T(t)(\boldsymbol{C}^T\boldsymbol{C}+\boldsymbol{K}^T\boldsymbol{RK})\boldsymbol{x}(t)\\ &\quad -\boldsymbol{e}^T(t)\boldsymbol{K}^T\boldsymbol{RKx}(t)-\boldsymbol{x}^T(t)\boldsymbol{K}^T\boldsymbol{RKe}(t)\\ &\quad +\boldsymbol{e}^T(t)\boldsymbol{K}^T\boldsymbol{RKe}(t)+((\bar{\boldsymbol{A}}-\boldsymbol{BK})\boldsymbol{x}(t)+\boldsymbol{BKe}(t)\\ &\quad +\bar{\boldsymbol{A}}_d\boldsymbol{x}(t-\tau))^T\boldsymbol{P}_x\boldsymbol{x}(t)\\ &\quad +\boldsymbol{x}^T(t)\boldsymbol{P}_x((\bar{\boldsymbol{A}}-\boldsymbol{BK})\boldsymbol{x}(t)+\boldsymbol{BKe}(t)+\bar{\boldsymbol{A}}_d\boldsymbol{x}(t-\tau))\\ &\quad +(-\boldsymbol{Fx}(t)+(\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})\boldsymbol{e}(t)\\ &\quad +\boldsymbol{MA}_d\boldsymbol{x}(t-\tau))^T\boldsymbol{P}_e\boldsymbol{e}(t)+\boldsymbol{e}^T(t)\boldsymbol{P}_e(-\boldsymbol{Fx}(t)\\ &\quad +(\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})\boldsymbol{e}(t)+\boldsymbol{MA}_d\boldsymbol{x}(t-\tau))\\ &\quad +\boldsymbol{x}^T(t)\boldsymbol{Q}_x\boldsymbol{x}(t)-\boldsymbol{x}^T(t-\tau)\boldsymbol{Q}_x\boldsymbol{x}(t-\tau)\\ &\quad +\boldsymbol{e}^T(t)\boldsymbol{Q}_e\boldsymbol{e}(t)-\boldsymbol{e}^T(t-\tau)\boldsymbol{Q}_e\boldsymbol{e}(t-\tau)\\ &\quad +\gamma^{-2}\boldsymbol{x}^T(t)\boldsymbol{P}_x\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^T\boldsymbol{P}_x\boldsymbol{x}(t)\\ &\quad +\gamma^{-2}\boldsymbol{e}^T(t)\boldsymbol{P}_e\boldsymbol{ME}\bar{\boldsymbol{E}}^T\boldsymbol{P}_x\boldsymbol{x}(t)\\ &\quad +\gamma^{-2}\boldsymbol{x}^T(t)\boldsymbol{P}_x\bar{\boldsymbol{E}}\boldsymbol{E}^T\boldsymbol{M}^T\boldsymbol{P}_e\boldsymbol{e}(t)\\ &\quad +\gamma^{-2}\boldsymbol{e}^T(t)\boldsymbol{P}_e\boldsymbol{MEE}^T\boldsymbol{M}^T\boldsymbol{P}_e\boldsymbol{e}(t)\\ &= \begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\\ \boldsymbol{x}(t-\tau)\\ \boldsymbol{e}(t-\tau)\end{bmatrix}^T \times \begin{bmatrix}\Pi_{11} & \Pi_{12} & \boldsymbol{P}_x\bar{\boldsymbol{A}}_d & \boldsymbol{0}\\ \Pi_{12}^T & \Pi_{22} & \boldsymbol{P}_e\boldsymbol{MA}_d & \boldsymbol{0}\\ \bar{\boldsymbol{A}}_d^T\boldsymbol{P}_x & \boldsymbol{A}_d^T\boldsymbol{M}^T\boldsymbol{P}_e & -\boldsymbol{Q}_x & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Q}_e\end{bmatrix} \times \begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{e}(t)\\ \boldsymbol{x}(t-\tau)\\ \boldsymbol{e}(t-\tau)\end{bmatrix}\end{aligned}$$

where

$$\begin{aligned}\Pi_{11} &= (\bar{\boldsymbol{A}}-\boldsymbol{BK})^T\boldsymbol{P}_x+\boldsymbol{P}_x(\bar{\boldsymbol{A}}-\boldsymbol{BK})+\gamma^{-2}\boldsymbol{P}_x\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^T\boldsymbol{P}_x\\ &\quad +\boldsymbol{C}^T\boldsymbol{C}+\boldsymbol{K}^T\boldsymbol{RK}+\boldsymbol{Q}_x\end{aligned}$$

$$\Pi_{22} = (\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})^T\boldsymbol{P}_e+\boldsymbol{P}_e(\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F}) +\gamma^{-2}\boldsymbol{P}_e\boldsymbol{MEE}^T\boldsymbol{M}^T\boldsymbol{P}_e+\boldsymbol{K}^T\boldsymbol{RK}+\boldsymbol{Q}_e$$

$$\Pi_{12} = \boldsymbol{P}_x\boldsymbol{BK}-\boldsymbol{K}^T\boldsymbol{RK}-\boldsymbol{F}^T\boldsymbol{P}_e+\gamma^{-2}\boldsymbol{P}_x\bar{\boldsymbol{E}}\boldsymbol{E}^T\boldsymbol{M}^T\boldsymbol{P}_e.$$

The sufficient condition satisfying the robust disturbance attenuation, $\sup_{d\in L^2} H[\boldsymbol{d}]<0$, is altered to fulfill the following matrix inequality:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \boldsymbol{P}_x\bar{\boldsymbol{A}}_d & \boldsymbol{0} \\ \Pi_{12}^T & \Pi_{22} & \boldsymbol{P}_e\boldsymbol{MA}_d & \boldsymbol{0} \\ \bar{\boldsymbol{A}}_d^T\boldsymbol{P}_x & \boldsymbol{A}_d^T\boldsymbol{M}^T\boldsymbol{P}_e & -\boldsymbol{Q}_x & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Q}_e \end{bmatrix}<0. \quad (19)$$

Moreover, Theorem III.1 transfers (19) into two algebraic Riccati inequalities using Shur decomposition and demonstrates the designs of $\boldsymbol{K}$, $\boldsymbol{L}$, and $\boldsymbol{F}$ which guarantee the robust disturbance attenuation.

Theorem III.1. Consider system (8) with the full-order compensator (10). If there exist the positive definite matrices $\boldsymbol{P}_x>0$, $\boldsymbol{P}_e>0$, $\boldsymbol{Q}_x>0$, and $\boldsymbol{Q}_e>0$ satisfying the following algebraic Riccati inequalities

$$\bar{\boldsymbol{A}}^T\boldsymbol{P}_x+\boldsymbol{P}_x\bar{\boldsymbol{A}}+\boldsymbol{Q}_x+\boldsymbol{C}^T\boldsymbol{C}+\boldsymbol{P}_x(\gamma^{-2}\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^T -\boldsymbol{BR}^{-1}\boldsymbol{B}^T+\bar{\boldsymbol{A}}_d\boldsymbol{Q}_x^{-1}\bar{\boldsymbol{A}}_d^T)\boldsymbol{P}_x<0 \quad (20)$$

$$\boldsymbol{X}_e\hat{\boldsymbol{A}}^T+\hat{\boldsymbol{A}}\boldsymbol{X}_e+\boldsymbol{X}_e(\boldsymbol{Q}_e+\boldsymbol{P}_x\boldsymbol{BR}^{-1}\boldsymbol{B}^T\boldsymbol{P}_x -\lambda\boldsymbol{C}^T\boldsymbol{C})\boldsymbol{X}_e+\boldsymbol{M}(\gamma^{-2}\boldsymbol{EE}^T+\boldsymbol{A}_d\boldsymbol{Q}_x^{-1}\boldsymbol{A}_d^T)\boldsymbol{M}^T<0 \quad (21)$$

where $\hat{\boldsymbol{A}}=\boldsymbol{MA}+\boldsymbol{F}$, $\boldsymbol{X}_e=\boldsymbol{P}_e^{-1}$, then robust disturbance attenuation (9) can be guaranteed. Furthermore, matrices $\boldsymbol{K}$, $\boldsymbol{L}$, and $\boldsymbol{F}$ are given by

$$\boldsymbol{K}=\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}_x, \quad \boldsymbol{L}=\frac{\lambda}{2}\boldsymbol{X}_e\boldsymbol{C}^T, \quad \text{and}$$

$$\boldsymbol{F}=\boldsymbol{M}(\gamma^{-2}\boldsymbol{E}\bar{\boldsymbol{E}}^T+\boldsymbol{A}_d\boldsymbol{Q}_x^{-1}\bar{\boldsymbol{A}}_d^T)\boldsymbol{P}_x,$$

Proof. By Shur decomposition, inequality (19) is equivalent to

$$\begin{bmatrix} \boldsymbol{J}_{11} & \boldsymbol{J}_{12} \\ \boldsymbol{J}_{12}^T & \boldsymbol{J}_{22} \end{bmatrix}<0 \quad (22)$$

where

$$\boldsymbol{J}_{11} = (\bar{\boldsymbol{A}}-\boldsymbol{BK})^T\boldsymbol{P}_x+\boldsymbol{P}_x(\bar{\boldsymbol{A}}-\boldsymbol{BK}) +\gamma^{-2}\boldsymbol{P}_x\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^T\boldsymbol{P}_x+\boldsymbol{C}^T\boldsymbol{C}+\boldsymbol{K}^T\boldsymbol{RK} +\boldsymbol{Q}_x+\boldsymbol{P}_x\bar{\boldsymbol{A}}_d\boldsymbol{Q}_x^{-1}\bar{\boldsymbol{A}}_d^T\boldsymbol{P}_x$$

$$\boldsymbol{J}_{12} = \boldsymbol{P}_x\boldsymbol{BK}-\boldsymbol{K}^T\boldsymbol{RK}-\boldsymbol{F}^T\boldsymbol{P}_e +\gamma^{-2}\boldsymbol{P}_x\bar{\boldsymbol{E}}\boldsymbol{E}^T\boldsymbol{M}^T\boldsymbol{P}_e+\boldsymbol{P}_x\bar{\boldsymbol{A}}_d\boldsymbol{Q}_x^{-1}\boldsymbol{A}_d^T\boldsymbol{M}^T\boldsymbol{P}_e$$

$$\boldsymbol{J}_{22} = (\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})^T\boldsymbol{P}_e+\boldsymbol{P}_e(\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F}) +\gamma^{-2}\boldsymbol{P}_e\boldsymbol{MEE}^T\boldsymbol{M}^T\boldsymbol{P}_e +\boldsymbol{K}^T\boldsymbol{RK}+\boldsymbol{Q}_e+\boldsymbol{P}_e\boldsymbol{MA}_d\boldsymbol{Q}_x^{-1}\boldsymbol{A}_d^T\boldsymbol{M}^T\boldsymbol{P}_e.$$

Designing $\boldsymbol{F}=\boldsymbol{M}(\gamma^{-2}\boldsymbol{E}\bar{\boldsymbol{E}}^T+\boldsymbol{A}_d\boldsymbol{Q}_x^{-1}\bar{\boldsymbol{A}}_d^T)\boldsymbol{P}_x$, $\boldsymbol{K}=\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}_x$, and $\boldsymbol{L}=\frac{\lambda}{2}\boldsymbol{X}_e\boldsymbol{C}^T$, then substituting them into (22) can attain

$$\boldsymbol{J}_{11} = \bar{\boldsymbol{A}}^T\boldsymbol{P}_x+\boldsymbol{P}_x\bar{\boldsymbol{A}}-\boldsymbol{P}_x\boldsymbol{BR}^{-1}\boldsymbol{B}^T\boldsymbol{P}_x+\boldsymbol{C}^T\boldsymbol{C} +\boldsymbol{Q}_x+\boldsymbol{P}_x(\gamma^{-2}\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^T+\bar{\boldsymbol{A}}_d\boldsymbol{Q}_x^{-1}\bar{\boldsymbol{A}}_d^T)\boldsymbol{P}_x$$

$$\boldsymbol{J}_{12} = \boldsymbol{0}$$

$$\boldsymbol{J}_{22} = \hat{\boldsymbol{A}}^T\boldsymbol{P}_e+\boldsymbol{P}_e\hat{\boldsymbol{A}}+\boldsymbol{P}_x\boldsymbol{BR}^{-1}\boldsymbol{B}^T\boldsymbol{P}_x-\lambda\boldsymbol{C}^T\boldsymbol{C} +\boldsymbol{Q}_e+\boldsymbol{P}_e\boldsymbol{M}(\gamma^{-2}\boldsymbol{EE}^T+\boldsymbol{A}_d\boldsymbol{Q}_x^{-1}\boldsymbol{A}_d^T)\boldsymbol{M}^T\boldsymbol{P}_e.$$

where $\hat{\boldsymbol{A}}=\boldsymbol{MA}+\boldsymbol{F}$. Therefore, if there exist $\boldsymbol{P}_x>0$, $\boldsymbol{P}_e>0$, $\boldsymbol{Q}_x>0$, and $\boldsymbol{Q}_e>0$ such that $\boldsymbol{J}_{11}<0$ and $\boldsymbol{J}_{22}<0$, it implies to $\sup_{d\in L^2} H[\boldsymbol{d}]<0$ and guarantees the robust disturbance attenuation. The proof of this theorem is completed. □

Remark III.2. Generally, for any γ for which a solution to (20) exists (which is used for the state feedback gain), we can find a λ large enough such that a solution to the inequality (21) exists. This means that a high gain compensator can be used to accomplish the work. Moreover, LMI technique [1] can be used to solve the two inequalities (20) and (21). Finally, we summarize the output feedback integral sliding mode controller as

$$\dot{\xi}(t) = (\boldsymbol{MA}-\boldsymbol{LC}+\boldsymbol{F})\xi(t)+(\boldsymbol{H}-\boldsymbol{FU})\boldsymbol{y}(t)$$

$$\boldsymbol{s}(t) = (\boldsymbol{GCB})^{-1}\boldsymbol{G}(\boldsymbol{y}(t)-\boldsymbol{y}(0)) +\int_0^t \boldsymbol{K}(\xi(q)-\boldsymbol{Uy}(q))dq$$

$$\boldsymbol{u}(t) = -\boldsymbol{K}(\xi(t)-\boldsymbol{Uy}(t))-\kappa(t)\frac{\boldsymbol{s}(t)}{\|\boldsymbol{s}(t)\|}.$$

IV. NUMERICAL EXAMPLE

To illustrate the proposed controller design, we consider the real example of chemical reactor system [6] and give the corresponding matrices for system (1) with delay time $\tau = 1$ as

$$\boldsymbol{A} = \begin{bmatrix} -4.93 & -1.01 & 0 & 0 \\ -3.20 & -5.30 & -12.8 & 0 \\ -6.40 & 0.347 & -32.5 & -1.04 \\ 0 & 0.833 & 11.0 & -3.96 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 & -0.2 & 0 \\ 0 & 1 & 0 & 0.1 \end{bmatrix},$$

$$\boldsymbol{E} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

and $\boldsymbol{A}_d = \text{diag}(1.92,\ 1.92,\ 1.87,\ 0.724)$. Moreover, unknown uncertainties and external disturbances for system (1) are set as

$$\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) = \begin{bmatrix} 0.12u_1 \sin t + 0.08u_2 \cos 1.3t + 0.2 \sin x_1 \\ 0.07u_1 \cos 3t + 0.03u_2 \sin 5t + 0.3 \cos x_2 \end{bmatrix}$$
$$\text{and} \quad d(t) = e^{-0.001t} \sin(2t)$$

where $\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$. The triple $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ has invariant zeros -4.4463 and -33.377, and $\text{rank}(\boldsymbol{CB}) = 2$. Choosing $\gamma = 0.2$, $\lambda = 10$, $\boldsymbol{G} = \boldsymbol{I}_2$, $\boldsymbol{R} = 2\boldsymbol{I}_2$, $\boldsymbol{Q}_x = \boldsymbol{I}_4$, and $\boldsymbol{Q}_e = 0.002\boldsymbol{I}_4$, we can calculate the solutions to (20) and (21) as

$$\boldsymbol{P}_x = \begin{bmatrix} 2.1171 & -0.028 & -0.4378 & -0.0497 \\ -0.028 & 2.0275 & 0.0076 & 0.2054 \\ -0.4378 & 0.0076 & 0.1190 & 0.0471 \\ -0.0497 & 0.2054 & 0.0471 & 0.1478 \end{bmatrix}$$

and

$$\boldsymbol{X}_e = \begin{bmatrix} 0.3554 & 0.0086 & 0.0169 & -0.0936 \\ 0.0086 & 0.3498 & -0.0116 & 0.0028 \\ 0.0169 & -0.0116 & 0.4623 & 0.1597 \\ -0.0936 & 0.0028 & 0.1597 & 0.6303 \end{bmatrix}.$$

Hence, the full-order compensator can be designed as

$$\dot{\xi}(t) = \begin{bmatrix} -3.1223 & 0.0846 & -5.9688 & 0.0043 \\ -0.0523 & -1.8338 & -1.0915 & 0.2143 \\ -6.4329 & 0.3833 & -31.6795 & 0.0177 \\ 0.6031 & 0.5054 & 10.8988 & -3.9262 \end{bmatrix} \xi(t)$$
$$+ \begin{bmatrix} -1.3621 & 0.0810 \\ 0.0025 & -0.0834 \\ -6.8105 & 0.4050 \\ -0.0246 & 0.8344 \end{bmatrix} \boldsymbol{y}(t)$$

$$\boldsymbol{v}(t) = \begin{bmatrix} -1.0585 & 0.0140 & 0.2189 & 0.0249 \\ 0.0140 & -1.0138 & -0.0038 & -0.1027 \end{bmatrix} \xi(t)$$
$$+ \begin{bmatrix} -1.0585 & 0.0140 \\ 0.0140 & -1.0138 \end{bmatrix} \boldsymbol{y}(t).$$

The sliding surface is designed as

$$\boldsymbol{s}(t) = \boldsymbol{y}(t) - \boldsymbol{y}(0) - \int_0^t \boldsymbol{v}(q)\, dq.$$

In order to avoid the chattering problem, the term $\frac{\boldsymbol{s}(t)}{\|\boldsymbol{s}(t)\|}$ in the control law can be replaced by the saturation function [24]. Therefore, the control input becomes

$$\boldsymbol{u}(t) = -\begin{bmatrix} 1.0585 & -0.0140 & -0.2189 & -0.0249 \\ -0.0140 & 1.0138 & 0.0038 & 0.1027 \end{bmatrix} \xi(t)$$
$$+ \begin{bmatrix} -1.0585 & 0.0140 \\ 0.0140 & -1.0138 \end{bmatrix} \boldsymbol{y}(t)$$
$$-\frac{1}{1-\chi}(\sigma_1 + \sigma_2 + \rho(t, y) + \chi\|\boldsymbol{v}(t)\|$$
$$+ \psi\bar{d} + \mu)\text{sat}(\boldsymbol{s}(t), \varepsilon)$$

where $\sigma_1 = \sigma_2 = 5$, $\rho = 2$, $\chi = \psi = 0.8$, $\bar{d} = 1$, $\mu = 2.5$, and $\text{sat}(\cdot)$ denotes the saturation function with $\varepsilon = 0.002$. Figures 1–5 chart the simulation results using the initial state $\boldsymbol{x}(0) = [2\ 3\ 4\ 1]^T$ and $\xi(0) = [0\ 0\ 0\ 0]^T$. The time responses of the system outputs are shown in Fig. 1. Figures 2 and 3 shows $\boldsymbol{s}$ and $\|\boldsymbol{s}\|$, respectively. In Fig. 3, the controlled system can maintain in the sliding layer in whole time. Figure 4 shows that the trajectories of $\boldsymbol{e}$ are bounded around zero and do not converge to zero because of the mismatch disturbance. In Fig. 5, the responses of the control inputs $\boldsymbol{u}$ are given, and the replacement of the saturation function eliminates the chattering. From Fig. 1, although the nominal system exists the state delay term and the mismatch disturbance, the system outputs $\boldsymbol{y}$ are finally bounded around zero. The simulation results demonstrate that the proposed controller design can guarantee the property of disturbance attenuation to outputs $\boldsymbol{y}$ once the system is in the sliding mode.

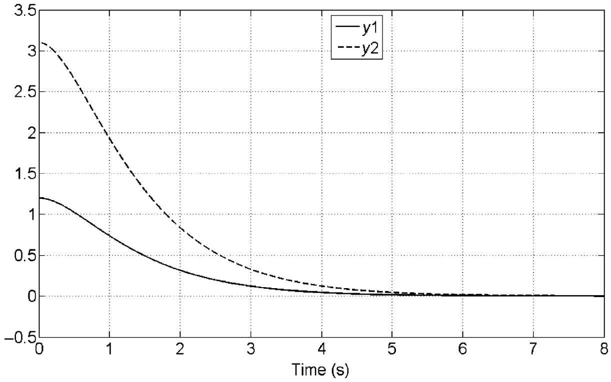

Fig. 1. System outputs.

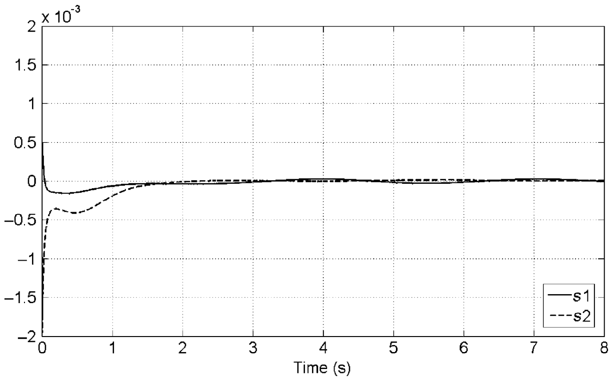

Fig. 2. Sliding surfaces.

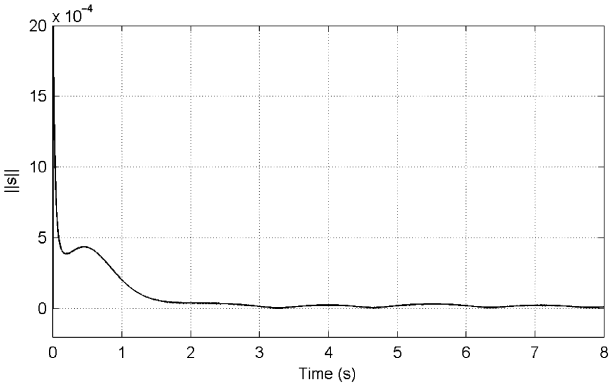

Fig. 3. Response of $\|s\|$.

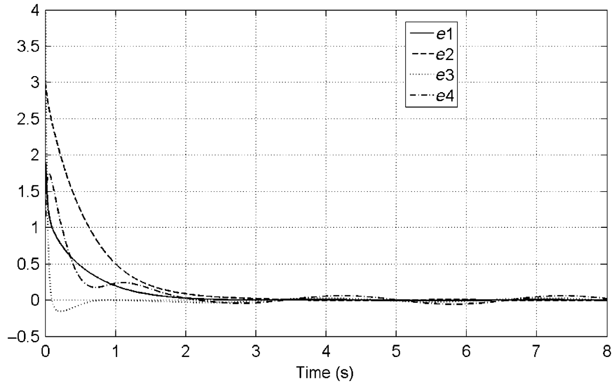

Fig. 4. Trajectories of $\boldsymbol{e}$.

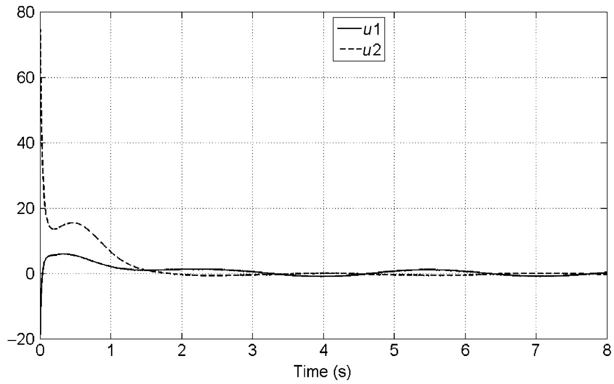

Fig. 5. System inputs.

V. CONCLUSIONS

This paper presents the output feedback integral sliding mode controller for a class of time-delay systems with mismatch disturbances. The auxiliary full-order compensator being added into the design of the integral sliding surface can improve the synthesis problem of static output feedback sliding mode control. We utilize the disturbance rejection condition in H_∞ theory to derive two algebraic Riccati inequalities comprised of the parameters of the system, controller, and compensator. When the two algebraic Riccati inequalities are satisfied, both the stability of the closed-loop system and the disturbance rejection condition can be guaranteed. Moreover, the designed controller can maintain that the system is always in the sliding mode from the initial moment. Finally, the simulation results of the real chemical reactor system demonstrated the feasibility of the propose control scheme.

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Huan-Chan Ting received his B.S. degree in Department of Electrical engineering from National Taipei University of Technology, Taipei, Taiwan in 2003. He is currently a candidate of Ph. D. degree in Department of Electrical and Control Engineering from National Chiao Tung University, Hsinchu, Taiwan. His major fields of research are sliding mode control theorem, robust control, and power electronics.

Jeang-Lin Chang received his B.S., M.S., and Ph.D. degrees in Department of Electrical and Control Engineering from National Chiao Tung University, Hsinchu, Taiwan. He is a professor in Department of Electrical Engineering, Orient Institute of Technology. His researches include variable structure control, robust control, and discrete signal processing.

Yon-Ping Chen received his B.S. degree in electrical engineering from National Taiwan University in 1981, and M.S. and Ph. D. degrees in electrical engineering from University of Texas at Arlington, U.S.A. He is a professor in Department of Electrical and Control Engineering, National Chiao Tung University. His researches include variable structure control, intelligent control, and image processing.